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The Alternating Current Commutator Motor.

The Leakage of Induction Motors.

By Dr.-Ing. R. GOLDSCHMIDT.

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THE ALTERNATING CURRENT COMMUTATOR MOTOR.

AND

THE LEAKAGE OF INDUCTION MOTORS.

BY

RUDOLF GOLDSCHMIDT,
Dr.-Ing.,

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INSCRIBED TO MY VENERATED TEACHER

DR. E. KITTLER

PROFESSOR OF ELECTRICAL ENGINEERING AT DARMSTADT COLLEGE.

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PREFACE.

When, some years ago, the single-phase motor with commutator made its appearance in the technical world, and its practical importance had been proved by successful designs, the attention of inventors was directed to this class of machines. A great many new types were introduced in all parts of the world, so that it was impossible for the average electrical man to follow and to distinguish between "improvement" and "new principle." Classification may, therefore, help to clear matters. One way of effecting this classification is to express every feature of the machines by their names. Another method, which has been preferred in this book, is to show that there are actually only two different principles :—

1. *Direct current machines, made suitable for alternating current* by laminating the field, inserting resistance between the armature and commutator, by compensating armature reaction, and by providing commutating poles wound in a special way.

2. *Repulsion motors* (I retain this name, which appears to be very popular, in spite of the fact that the word "repulsion" is not strictly correct). The quality of alternating current energy, that it can be transmitted from one circuit to

another without electrical connection, has been made use of by "inducing" the current in the armature instead of "conducting" it there.

All other inventions are modifications of these principles, the most important being the use of the armature as an exciting winding (Latour, Winter-Eichberg), a method by which the power factor is improved at the same time, and enables us to make a constant speed motor.

This book has been written for the engineer who has a thorough knowledge of direct current machines and the principles of alternating current machines, and also for the specialist, who will find some original treatises (the core loss and circulating current question, &c.) fully dealt with.

I have to thank the different firms who have kindly supplied me with photographs or blocks of their machines.

R. GOLDSCHMIDT.

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ERRATA.

The following small errors have been detected in the text of Dr. Goldschmidt's Book, "Alternating Current Commutator Motors":—

Page 26: Equation (10), 2nd line, should read—

$$=\frac{1}{190}\times\frac{n \, . \, p}{v}\times\frac{1}{c}\sqrt{\frac{1}{\cos^2\phi}-1}. \quad . \quad . \quad . \quad (10)$$

Page 114: 14th line, Fig. 124; 15th line, Fig. 125; 21st line, Fig. 122.

Page 124: 11th line, Section 5, *instead of* Chapter V.

THE ALTERNATING CURRENT COMMUTATOR MOTOR.

CHAPTER I.

INTRODUCTION.

1. Elementary Electro-Dynamic Laws.

To study thoroughly the function of commutator motors worked with alternating current, it is well to go back to the fundamental electro-dynamic laws relating to a single conductor in a magnetic field. Every alternating current and every alternating field are, considered for a very short interval of time, a constant current and a constant field, at least as far as their interaction, the production of torque and its electrical equivalent, the production of E.M.F. through cutting of lines by the moving conductor, are concerned.

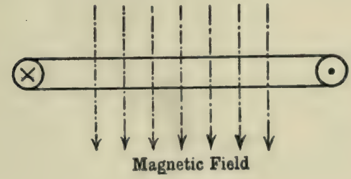


FIG. 1.

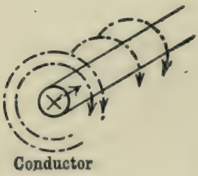


FIG. 2.

Fig. 1 represents a magnetic field of a certain density B, Fig. 2 a portion of the conductor carrying the current I.

The field in Fig. 1 may be produced by some large coil; it may be a portion of the field in the air-gap of an electromotor; it does not matter how we have produced it.

Inseparable from our current-carrying conductor in Fig. 2 are magnetic lines of force closed round the lead. Looking at a section of the latter and imagining the current to flow away from us, the direction of the magnetism will be clock-wise.

If we place the conductor in the magnetic field, the latter will lose its uniformity, the lines round the conductor combining with the original ones in a way, drawing them to one side like a heavy curtain. The curtain of lines tending to go back to its original position moves the conductor along in the direction of the feathered arrow (Fig. 3).

Bearing in mind this law, together with that of the direction of the flux around the conductor, we can easily find the direction of rotation of any machine. If, for instance, Fig. 4 represents diagrammatically a direct current machine, and the magnetic action of the armature conductors lifts the curtain of lines to the right, they will drag the conductors to the left, *i.e.*, the rotation of the machine will be counter-clockwise. The well-known backward movement of the neutral magnetic zone of the armature is quite obvious, as, following our picture, the curtain must be drawn *back* so as to shift the armature *forward*.

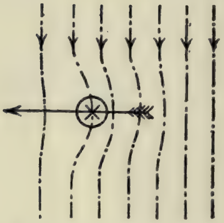


Fig. 3.—Conductor in Magnetic Field.

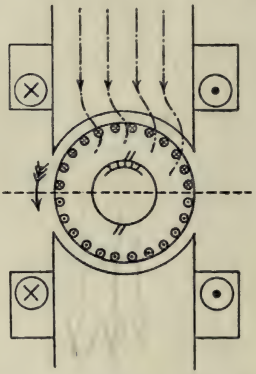


Fig. 4.—Section through Commutator Motor.

If the conductor in Fig. 3 and the armature in Fig. 4 follow the mechanical pull exercised on them, mechanical work is given out and equivalent electrical energy has to be produced.

During the movement of the conductor magnetic lines are cut and an E.M.F., E, induced, opposing the current and thus trying to reduce its value. An E.M.F. equal to E has to be spent from an outside source so as to keep up the current against the counter-E.M.F. due to the movement in the magnetic field. The electrical energy, E × I, is the electrical equivalent for the mechanical work done.

We do not want to go further into these elementary details. The object of mentioning them at all was to come into touch with the fundamental laws. With their help we can decide the direction of rotation in complicated cases. In the case of two or more fields acting on an armature, or two or more armature currents being in a field, we shall be able to determine whether the actions add together or subtract from each other, &c. Furthermore, it is well to remember in complicated cases that to determine the mechanical energy of a motor it is only necessary to find the electrical equivalent, the counter-E.M.F. due to the movement of the conductors, and to multiply it by the current.

2. The Introduction of the Commutator.

A conductor at the surface of an armature carrying a certain current produces a torque, say, in a counter-clockwise direction when moving in the sphere of influence of a certain pole, say the pole from which the magnetic lines enter the armature (Fig. 5).

If the conductor leaves this sphere and, passing the neutral line A, B, enters the region where the magnetic lines leave the armature, the torque exercised on the conductor reverses, as can be readily seen, if we apply the curtain rule. This holds good for both alternating and direct current, if with the former we consider only a very short interval of time. To make the armature revolve, the direction

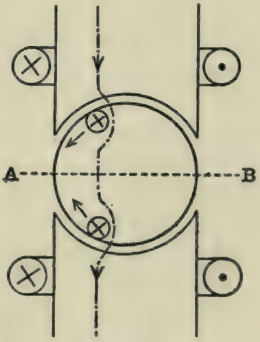


FIG. 5.

of current must change when the conductor passes the neutral zone A, B. This can be achieved by making the current in the armature an alternating one, having such a frequency that it changes its sign when the conductor passes through the neutral zone. This changing of sign of an alternating current in the neutral zone might be called "natural" or "electrical" commutation. As the frequency of an alternating current supply, available for driving motors, must be considered as something absolutely fixed, the speed of the motor must adapt itself to it. There is only one speed, the synchronous speed, at which the machine can work. It cannot—and this is important—work at the speed "zero"—*i.e.*, it cannot start by itself—the great disadvantage of synchronous motors which excludes them from all work requiring a certain amount of torque at starting.

A second means to change the direction of the current, as soon as the lead passes through the neutral zone, is the commutator and brushes, as indicated in Fig. 4, and as universally applied to continuous current machinery—the "artificial" or "mechanical" commutation.

The ends of every conductor or group of conductors round the armature are led to segments of the commutator, on which brushes are slipping. These conduct the current into the armature, but as the latter revolves together with the commutator, and the brushes are placed accordingly, the current in every conductor is reversed as soon as it passes the neutral zone. The great advantage of the commutator method of reversing the current in the "neutral" is that such machines are workable at any speed. Whatever the working conditions may be, and whatever speed the machine may run at, independently of its being supplied with alternating or direct current, the commutator does its duty—namely, to change the direction of the current in the armature leads as soon as the latter pass from the region of one pole into that of the next. If, therefore, the working conditions of armature and field under the first pole have been satisfactory, they will be so under the next one following. It is this quality which makes the commutator motor desirable and which has contributed to make the leading engineers work out alternating current commutator motors for application to railway work and general industrial use, in spite of the enormous difficulties to be overcome.

To complete our study we will mention that in the case of polyphase induction motors (asynchronous motors) the change of the sign of the armature current in the neutral zone—*i.e.*, the zero point of the field, in this case a revolving one—takes place through alternating current flowing in the rotor just as with the synchronous motor, *i.e.*, through "natural" commutation. The induction motor has the advantage that the armature current is not taken from the supply, but is induced through the revolving field itself, and that its periodicity changes as the speed changes, in such a way that the direction of the current alters automatically as soon as a rotor conductor passes out of the range of one pole into that of the next. The starting of the polyphase motor is good, but it has the drawback that efficient working can only take place very near synchronous speed.

3. Counter E.M.F. and Torque.

We will now pass on from the elementary considerations of a single conductor in a field and direct our attention to armatures built exactly as those in use with continuous current machines—*i.e.*, drum winding closed in itself and arranged in slots at the outer circumference of the laminated cylindrical iron core. The ends of the single armature coils are connected to commutator segments. Assuming we have a two-polar machine, two brushes diametrically opposite one another slide on the commutator and conduct the current into the armature, dividing at the same time the armature winding into two halves, which carry current in opposite directions. Fig. 4 diagrammatically gives a picture of the current distribution, in a section of a conductor marked ⊗ the current flowing away from us, in one marked ⊙ the current flowing towards us.

This diagram of the current distribution is true for every position of the armature.

To calculate the torque and the power we have, as explained above, to use the law for the E.M.F. induced in the armature through its motion in the magnetic field, the equivalent for the speed. We put the brushes into the neutral position and assume the magnetic flux issuing from one pole and entering the other to have the value N expressed in C.G.S. lines. If we call further

s_a the number of conductors all round the armature,
n the speed in revs. per min.,

then the E.M.F. induced in the armature through rotation is, according to the well-known formula in use for direct-current machines,

$$E = s_a \times \frac{n}{60} \times N \times 10^{-8} \text{ volts}$$
$$= \text{Total conductors} \times \text{revs. per sec.} \times \text{flux} \times 10^{-8}. \quad . \quad (1)$$

It is necessary to emphasize the fact that the distribution of the flux around the armature has no influence on this E.M.F. In Fig. 6A the conductors on one side of the armature have

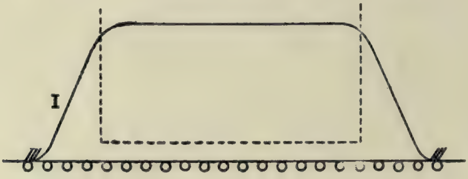


Fig. 6A.

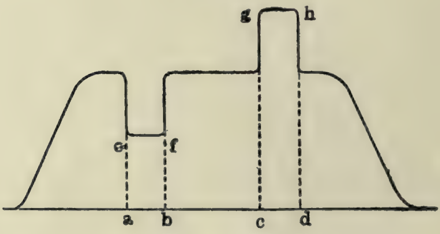


Fig. 6B.

been spread out in a straight line and the distribution of the flux has been shown by a diagram. At the "neutral" the flux density is zero, rising up to a certain maximum under the pole and remaining there practically constant. (Curve I.) Let us remove from part of the armature surface (ab) a portion of the flux, reducing the density here to $ae=bf$, and transfer this flux to another part cd (so that $cd=ab$), increasing here the density the same amount we reduced it on ab (Fig. 6B). The total flux thus remains the same as before. The result will be that the conductors between c and d receive exactly as much more E.M.F. as those from ab receive less. The total E.M.F. will be exactly the same as with the distribution in Fig. 6A.

The law that the form of the flux curve has no influence whatever on the E.M.F., and that the total flux N fixes the value of the latter, is important with alternating current motors, because widely different flux distributions are in practical use. With a direct current machine we generally—a few recent constructions (turbo-generators) excepted—have one field coil per pole surrounding the latter, and the pole itself ends in a pole-shoe with a face giving an even air-gap between pole and armature. Sometimes a slight bevel of the pole corner is used. Our curve in Fig. 6A is representative of the flux distribution in such a case. With alternating current commutator motors, too, this form is much used, but one finds also machines which have no projecting poles at all. The stationary portion is a cylinder surrounding the armature quite uniformly,

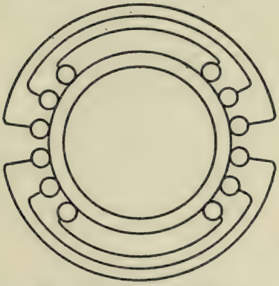


Fig. 7.—Distributed Field Winding.

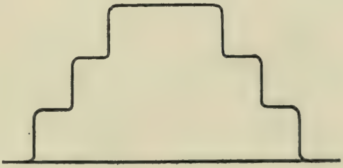


Fig. 8.—Field Curve with Distributed Field Winding.

and the field winding consists of several concentric coils embedded in slots on the inner surface of the cylinder. The whole arrangement resembles that of an induction motor very closely (Fig. 7). With such a machine the flux assumes the form of Fig. 8. We have taken the number of slots per pole to be six, one-half of a pole pitch in the centre of the coils being free from winding, as here the opening of the coils would become too small to be effective. It will be our task presently to find the most economical way of excitation; for the present we shall disregard the method of producing the flux altogether.

At every moment the E.M.F. is fixed by formula (1); *therefore, if the flux* N *is an alternating one, the E.M.F., following it very closely in every respect, is also alternating.* E *is always proportional to* N, *has the same form expressed as a function of the time and the same periodicity, regardless of the speed.*

Taking the exciting current to alternate sinusoidally, and disregarding saturation for the present N, and consequently E, will follow the same law as the current.

At the moment where N has its maximum value $N_{max.}$, we have

$$E_{max.} = s_a \times \frac{n}{60} \times N_{max.} \times 10^{-8} \text{ volts,}$$

and the root mean square value

$$E = \frac{1}{\sqrt{2}} \times s_a \times \frac{n}{60} \times N_{max.} \times 10^{-8} \text{ volts.} \quad . \quad . \quad (2)$$

It is usual to introduce $N_{max.}$ into the formula for E, as the maximum of the flux density fixes the saturation of the iron and the core loss, and is limited by the flux-carrying capacity of the iron and by heating.

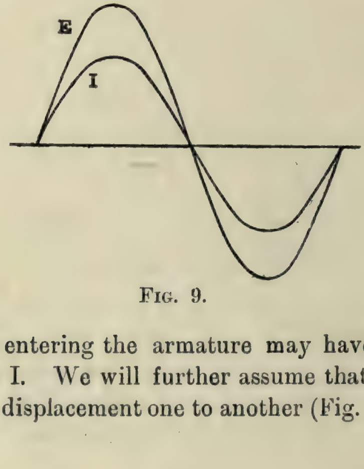


Fig. 9.

The current entering the armature may have the effective value (R.M.S.) I. We will further assume that I and E shall have no phase displacement one to another (Fig. 9). Then the electrical power converted into mechanical power is

$$P = E \times I \text{ watts} = E \times I/746 \text{ H.P.}; \quad . \quad . \quad . \quad (3)$$

the torque in pound-feet is

$$T = 5{,}250 \times \frac{\text{horse-power}}{\text{revs. per min.}} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Introducing the value for E we find

$$P = \frac{1}{\sqrt{2}} \times s_a \times \frac{n}{60} \times I \times N_{\text{max.}} \times 10^{-8} \text{ watts},$$

$$= \frac{1}{63{,}400} \times s_a \times n \times I \times N_{\text{max.}} \times 10^{-8} \text{ H.P.}$$

Torque $T = \frac{1}{12} \times s_a \times I \times N_{\text{max.}} \times 10^{-8}$ lb.-ft.

$$= \tfrac{1}{6} \times \text{ampere-conductors on armature} \times \text{max. flux} \times 10^{-8} \text{ lb.-ft.} \quad . \quad . \quad . \quad (5)$$

The torque calculated is naturally not the actual torque available on the shaft. We should have to deduct the hysteretic and friction momentum, amounting to 5 to 10 per cent. of the total. This will be determined later on, as it does not affect the principle of working of the commutator motor.

With a direct-current machine the factor $1/\sqrt{2}$ in the formula for P would drop out, so that with the same $N_{\text{max.}}$ and the same R.M.S. value for the current, the output of an alternating current commutator motor is 29½ per cent. less than that of a direct-current machine.

Our assumption that the counter E.M.F. of the motor is in phase with the current is identical with the condition that current and magnetic flux are in phase, and, with an ideal motor free from hysteresis, that exciting and armature currents are in phase.

As a general expression for the electrical power we have to put

$$P = E \times I \times \cos\alpha, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where α is the angle of phase displacement between I and E (Fig. 10). The same factor, $\cos\alpha$, appears in the formula for the torque

$$T = \tfrac{1}{12} \times I \times s_a \times N_{\text{max.}} \times \cos\alpha \times 10^{-8} \text{ lb.-ft.}$$

If α is 90 deg. the machine cannot work, no torque being produced. We will consider this case a little more in detail.

Fig. 12 shows the two sine curves of field N and armature current I in the case of 90 deg. phase displacement. Consider a quarter period A–B during which the flux is positive, the current is negative, and the torque exercised on the conductor in Fig. 11 has the direction to the right. When the flux is at its height (at B) the current changes its sign, and during the quarter period (B–C) the torque is reversed (Fig. 11A). Plotting the torque curve in Fig. 12 we find it to be a sine curve

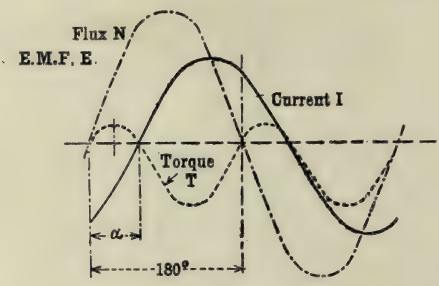


Fig. 10.

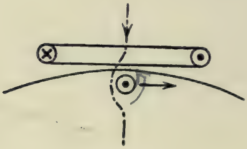


Fig. 11.

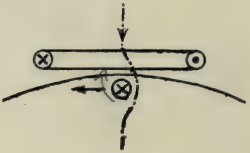


Fig. 11A.

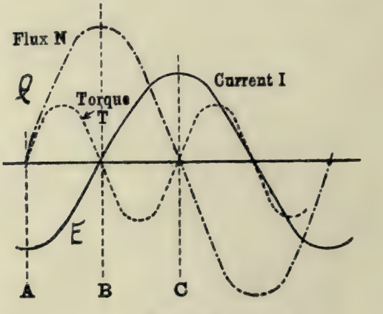


Fig. 12.

with twice the periodicity of that of the current. The average torque is zero.

Fig. 10 shows the torque curve for α degrees displacement of flux and current.

4. The Production of the Magnetic Field.

The general behaviour of the alternating current commutator motor would be very much the same as that of a direct current machine if an alternating current magnetic field could be produced as simply and inexpensively as a direct-current field. In both cases, to drive a flux through field, air-gap and armature a certain number of ampere-turns in the field coils are required; the same with alternating and direct current motors. There is some slight distinction necessary, due to our calculations being made with the maximum flux $N_{max.}$

Suppose that, to produce a constant flux equal to $N_{max.}$, a direct current, i_c, must flow in the z_f field-turns. The ampere-turns being $i_c \times z_f$, then the *maximum* alternating current excitation must be equal to i_c. Hence, the effective value (R.M.S.) of the exciting current, taking the curve form to be sinusoidal, is

$$\dot{i}_f = \frac{1}{\sqrt{2}} \cdot \dot{i}_c.$$

This difference, naturally, is only due to our using $N_{max.}$ Introducing directly the E.M.F. induced in the armature, the factor $1/\sqrt{2}$ drops out as it occurs in the formula (2) for E as well, and we can say that, *to produce in a given armature at a fixed speed a certain E.M.F. (R.M.S.), the same exciting current in the field is required, regardless of its being direct current or alternating, provided that the iron is not saturated.*

With direct and alternating current alike the current flowing through the field coils causes the same ohmic losses in its resistance and the same ohmic drop, provided the coils are otherwise the same. But the flux has ceased to be constant, and the well-known effect of the alternating flux makes itself felt—*i.e.*, (1) the iron resists the variation of the lines, hysteresis and eddy-current losses being caused. *This necessitates lamination of the field iron in alternating motors.* (2) The alternating flux, passing through the field coils, induces an E.M.F. in them.

These phenomena are just the same as those occurring in all alternating apparatus, such as transformers, choking coils, &c.

The question of the core losses can be dealt with in the well-known way. Knowing the density in the iron and the quality of the latter, the calculation of the core losses in the field does not offer greater difficulty than, for instance, in an induction motor. To calculate the core loss in the armature is slightly more complex, because of the rotation of the latter, making it worth while to go more carefully into this question. We shall, however, defer this to a later chapter, and devote ourselves entirely to the study of the inductive effects. In doing so we shall neglect for the present entirely the losses and drops caused through hysteresis and the ohmic resistance of the coils. We shall assume ideal field coils without ohmic resistance, and iron carrying the varying magnetism without loss and without being saturated.

It will generally be our policy, in cases of very complicated questions, to treat ideal cases first, omitting comparatively minor quantities which do not materially affect the problem in question. They would only blur the clear picture of the principal subject if we took account of them from the first. After having developed the main points, it is very easy to allow for all corrections afterwards.

If z_f denotes the number of field turns, v the frequency in periods per second of the supply, and naturally also the frequency of flux and field current, further $N_{f\,\text{max.}}$ the amplitude of the magnetic flux, then the formula for the E.M.F. induced in the field coil is

$$E_L = 4{\cdot}44 \times z_f \times v \times N_{f\,\text{max.}} \times 10^{-8} \text{ volts.} \quad . \quad . \quad (3)$$

This is the well-known formula for the E.M.F. caused by an alternating flux passing through a number of turns, as used with all alternating apparatus. To keep up the flux $N_{f\,\text{max.}}$, we have to supply the field coil with a voltage e_L, equal and opposite in phase to E_L. Further, the magnetising current i_f is flowing; e_L and i_f are at 90 deg. phase displacement, and the product $e_L \times i_f$ does not represent any energy. The current is wattless, the same as with a choking coil. The energy stored in fields and field coils during the period of creating the flux is given back a moment later when the field disappears. Represented in a linear diagram (Fig. 13) the sine curve i_f is the field current, being in phase with the flux N_f, and, created by it, the sine wave for the induced E.M.F. E_L follows flux and current by 90 deg. To overcome E_L, the E.M.F. e_L must be equal and opposite to E_L. Fig. 14 represents the same phase relation in a vector diagram.

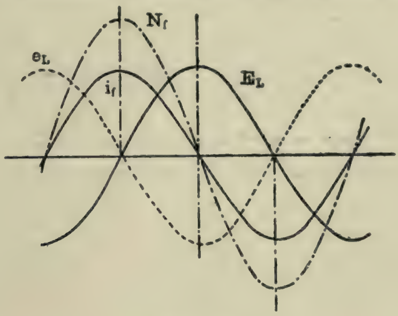


FIG. 13.

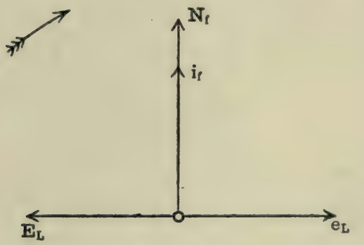


FIG. 14.

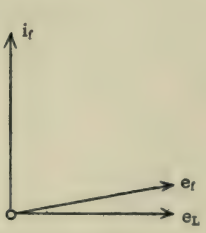


FIG. 15

The losses in resistance and core affect this diagram only to a very small degree. They reduce the phase displacement between the terminal voltage of the field coil, which we will call e_f, to distinguish it from its inductive component e_L (Fig. 15). With practical motors the cosine of the angle between e_f and i_f is about 0·10 to 0·15.

Though, except for the comparatively small losses, actually no power is consumed, *the wattless voltage and current must be taken from the line.* $e_L \times i_f$ volt-amperes must actually be produced by the generator. Calling this amount of wattless power $P_{Lf} = E_L \times i_f$, we can express it by the following formula :—

$$P_{Lf} = 4{\cdot}44 \times i_f \times z_f \times N_{f\,\mathrm{max.}} \times v \times 10^{-8} \text{ volt-amperes}, \quad . \quad (4)$$

or wattless power consumed by field $= 4{\cdot}44 \times 10^{-8} \times$ ampere-turns × max. flux × periodicity.

The ampere-turns required to produce $N_{f\,\mathrm{max.}}$ are proportional to $N_{f\,\mathrm{max.}}$ itself and to the reluctance R of the magnetic path. Putting

$$\text{Ampere-turns} \times \sqrt{2} = \frac{10}{4\pi} \times R \times N_{f\,\mathrm{max.}}\,; \text{ and } N_{f\mathrm{max.}} = \frac{\frac{4\pi}{10} \times \text{amp.-turns} \times 1{\cdot}41}{R}$$

we can say

$$P_{Lf} = 2{\cdot}5 \times 10^{-8} \times N^2_{f\,\mathrm{max.}} \times R \times v,$$

or

$$P_{Lf} = 7{\cdot}8 \times \frac{(i_f \times z_f)^2}{R} \times v \times 10^{-8}$$

$$= 7{\cdot}8 \times 10^{-8} \times \frac{(\text{ampere-turns})^2}{\text{reluctance of magnetic path}} \times \text{periodicity}. \quad . \quad (5)$$

The similarity to Joule's law will be noticed. These formulæ tell us that *the wattless power to produce the magnetic field is proportional to the periodicity and the reluctance of the motor, and proportional to the square of the magnetic flux.* A small air-gap reducing the reluctance and a low periodicity are, therefore, desirable.

Our flux N_f would be identical with the flux in the armature N, except for the leakage lines. Just the same as with direct current machinery, additional flux lines find their way through leakage paths round the field coils. The leakage coefficient—*i.e.*, the ratio N_f/N—is about 1·1 or 1·15 or smaller, dependent on the design. Calling the leakage co-efficient c, we can write the formulæ for E_L and P_{Lf}

$$E_L = 4{\cdot}44 \times c \times z_f \times v \times N_{\mathrm{max.}} \times 10^{-8}. \quad . \quad . \quad . \quad (6)$$

$$P_{Lf} = 4{\cdot}44 \times c \times (i_f \times z_f) \times N_{\mathrm{max.}} \times v \times 10^{-8}. \quad . \quad . \quad (7)$$

The basis for these formulæ has been an arrangement as shown in Fig. 16, the field coils embracing the whole of the magnetic flux. If the field winding is split up into several smaller coils distributed in slots on the inner circumference of the stator, the flux passes through between the different coils. It is not clear at first sight if such an arrangement offers an advantage over that with concentrated coils—*i.e.*, if, to produce a certain flux, less wattless power is necessary in one case or the other. We can easily study this question by starting from the arrangement with concentrated windings (Fig. 17) and splitting one-half of the coil off, placing it in two slots, *c* and *d* (Fig. 18). Both coils are concentric, coil 2 having a smaller span, x, than coil 1, the span of the latter being y (Fig. 18). $z_f/2$ is the

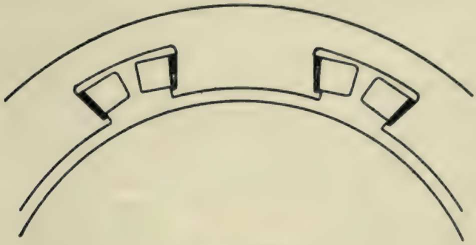


FIG. 16.

FIG. 17.

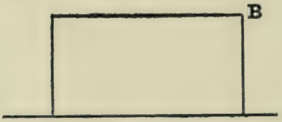


FIG. 17A.—FIELD FORM WITH CONCENTRATED WINDING.

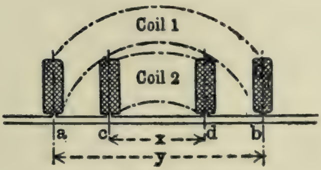


FIG. 18.

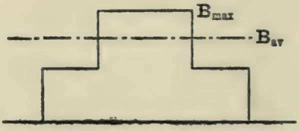


FIG. 18A.—FIELD FORM WITH DISTRIBUTED WINDING.

number of turns per coil. The portions *a-c* and *d-b* of the field are only exposed to the magnetising power of one-half of the total ampere-turns; the diagram of the flux density, therefore has the form Fig. 18A, the average density, $B_{av.}$, being

$$B_{av.} = \frac{B_{max.}}{2} + \frac{B_{max.}}{2} \times \frac{x}{y} = \frac{B_{max.}}{2}\left(1 + \frac{x}{y}\right).$$

To obtain the same flux as in Fig. 17, $B_{.}$ must be the same, and, therefore, the exciting current must be increased in the proportion

$$\frac{2}{1 + x/y}.$$

For $x/y = \frac{1}{2}$ the ratio is 1·33, and 33 per cent. more amperes would be required to produce the same flux. But in spite of the increase in ampere-turns, the watts may be less, as they depend on E_L also. We find E_{L2} for the inner coil, considering that $B_{max.}$ is $\frac{2}{1 + x/y}$ higher than $B_{av.}$,

$$E_{L2} = \frac{1}{1 + y/x} \times E_L.$$

The total flux remaining constant, E_{L1} must be one-half times E_L and therefore

$$E_{L1} + E_{L2} = \left(\frac{1}{2} + \frac{1}{1 + y/x}\right) E_L.$$

For $x/y = \frac{1}{2}$, $E_{L1} + E_{L2} = 0{\cdot}83 \times E_L$—*i.e.*, the E.M.F. of induction is 17 per cent. smaller than before. Calling P'_{Lf} the wattless power with arrangement Fig. 18, we find

$$P'_{Lf} = \frac{1 + 3 \times x/y}{(1 + x/y)^2} \times P_L.$$

P'_{Lf} is a maximum for $x/y = \frac{1}{3}$, and for this figure the wattless energy is $P'_{Lf} = 1{\cdot}125 P_L$—*i.e.*, 12½ per cent. higher than if the coils were concentrated. The curve Fig. 19 shows the ratio P'_{Lf}/P_{If} as the function of x/y.

An ideal limiting case is worth considering—viz., the uniform distribution of the field winding over the whole inner surface of the field. This case is important because sometimes the exciting current has to flow in the distributed winding of the armature itself, as will be seen later on. The curve for the density is then triangular (Fig. 20). Considering the total flux, and, therefore, $B_{av.}$ as fixed, $B_{max.}$ is $2 \times B_{av.}$ The exciting current with the same number of turns is, therefore, double that of the concentrated winding. The flux N_x, comprised by a turn with the span x, is a measure of the E.M.F. induced in it, or

$$E_x \propto B_{max.} \times x\left(1 - \frac{x}{2y}\right).$$

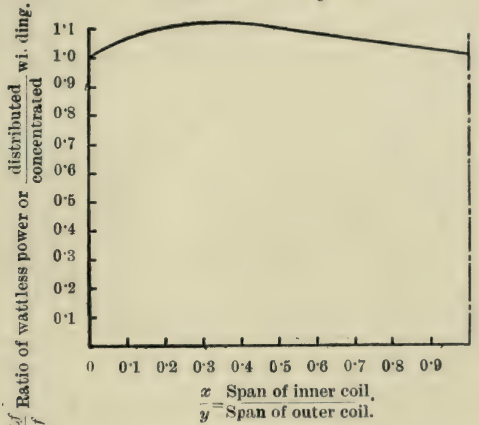


FIG. 19.

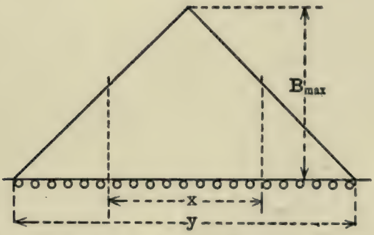


FIG. 20.—FIELD FORM WITH FLAT WINDING.

The average E.M.F. per turn is, therefore,

$$\frac{B_{max.}}{y} \times \int_o^y \left(x - \frac{x^2}{2y}\right) dx = \frac{B_{max.}}{3} \times y.$$

In the same measure the average E.M.F. for the turn of a concentrated coil would be $\frac{B_{max.}}{2} \times y$. The E.M.F. induced in the distributed winding by a certain flux is, therefore, only one-third that of a concentrated one. The current being double, the distributed winding consumes 50 per cent. more wattless energy. This 50 per cent. is partly made up by reduced leakage. As a general conclusion we may say *the wattless energy necessary to produce a certain flux is smaller the more the field winding is concentrated.*

The ohmic loss in the field varies together with *the ampere-turns*, and the latter *are smaller the greater the concentration of the field winding*, this effect not being compensated for by the increase of mean length of turn.

The core losses in teeth and poles are also smaller the more the ratio $B_{max.}/B_{av.}$ approaches unity, the latter being reached practically with a one-coil winding.

5. The Simplest Forms of Excitation: Shunt and Series Motor.

The most frequent form of a direct current machine is the shunt motor. We might almost call it the most natural way to supply the exciting coils with their energy separately from the armature. Further, this constant-speed form of machine is most suitable for the majority of practical purposes. Let us consider the qualities of such a machine, when supplied with alternating current.

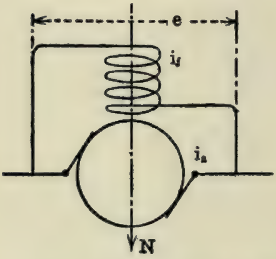


FIG. 21.—SHUNT MOTOR.

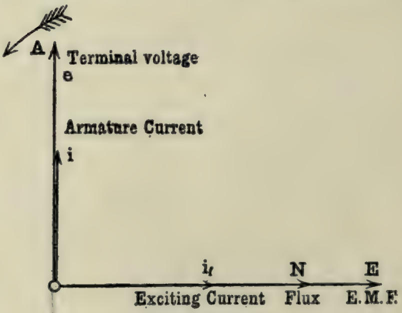


FIG. 22.

Fig. 21 shows a diagram of such a machine. The terminal voltage of the motor being the terminal voltage of the field, the exciting current i_f and the flux N_f will be displaced in phase from e by almost 90 deg. This phase relation is borne out by Figs. 13, 14 and 15, and also by diagram Fig. 22;

OA is *e*, OC the flux N, 90 deg. behind *e*. The armature revolving in the flux N consequently generates the counter E.M.F. E, which, as is easily seen, is exactly in phase with flux N, and, therefore, lagging 90 deg. behind *e*. On the other hand E must be counterbalanced, it must be supplied as a component of the terminal voltage *e*, but both being in 90 deg. phase displacement, *e cannot* have a component in phase with E. We can, therefore, conclude at once *that the shunt motor in this form cannot work*. There are means by which such a shunt machine can be made to develop torque; for instance, this can be effected by bringing the phases of N, with its E and *e*, closer together. Ohmic resistance may be introduced in the field winding, thus shifting the phase of i_f, N and therewith E closer to *e*. At the same time self-induction must be introduced in the armature circuit, possibly by increasing the natural re-actance of the armature, so as to make the armature current lag behind the terminal voltage (Figs. 23 and 24). It is evident that these expedients are not practical ones; the resistance in the field coils would consume an enormous amount of energy. As stated above, the power factor of the field circuit is ordinarily about 0·10 to 0·15. To bring the flux within 45 deg. of *e*—*i.e.*, to produce a power factor of the field equal to 0·71, the losses in the field circuit must be about six

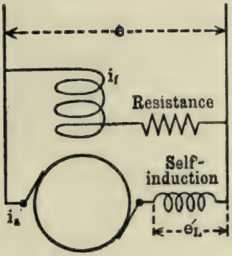


FIG. 23.—SHUNT MOTOR WITH CHOKER AND RESISTANCE.

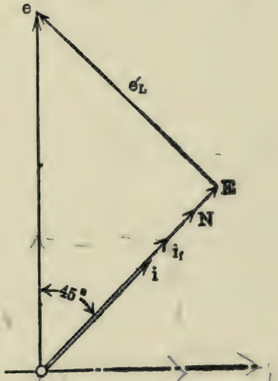


FIG. 24.

times the ordinary losses, which means a reduction in efficiency of something like 20 to 25 per cent. In trying to make up the rest of the phase displacement between *e* and E—45 deg.—an E.M.F. of self-induction must be introduced in the armature circuit, just as big as the useful counter E.M.F. E itself. The power factor, therefore, will be very low indeed.

We shall see later that there are motors with "shunt" characteristics, but these are machines of a quite special construction, entirely different from that of the direct current shunt motor.

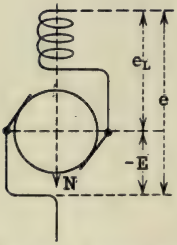


FIG. 25.—SERIES MOTOR.

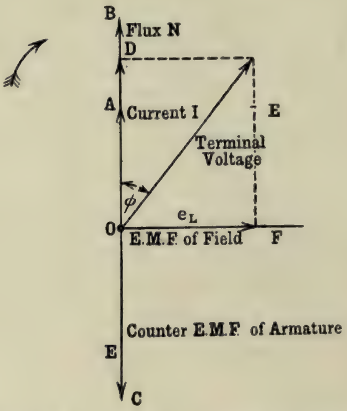


FIG. 26.—DIAGRAM OF SERIES MOTOR.

We have come to these conclusions very quickly, on the basis that the terminal voltage must have a component in phase with the counter E.M.F. to make the motor work. This supposition is the same as if we had said that the current must have a component in phase with the flux as already explained. The motor best suited for alternating-current work is a machine whose flux and armature current are in phase, *i.e.*, whose counter E.M.F. and terminal voltage of the armature are in phase, without it being necessary to introduce an E.M.F. of self-induction (e'_L in Fig. 24) to adjust the lack of phase coincidence. This condition is excellently fulfilled in the plain series motor. Fig. 25 shows diagrammatically such a motor. The current naturally is the same in field and armature, and thus also the magnetic flux produced by the field coil and the current in the armature are—apart from hysteretic lag—in phase. Due to the hysteresis component of the current, and other causes which we shall consider later, some slight phase displacement between flux and current exists. For the present we will neglect this amount. Diagram Fig. 26, therefore, becomes very simple. OA is the current, in phase with it the flux OB = N, and in phase with the flux the counter E.M.F. OC = E. If we have—as we will assume for the present—no self-induction in the armature and no resistance, the terminal voltage of the armature would have only to balance E, and would present itself as a vector OD = − E. We find from diagram Fig. 14 the terminal voltage of the field e_L to be 90 deg. in advance of the current, and represented by the vector OF; these E.M.Fs., − E and e_L, together equal the terminal voltage of the machine e.

We see that the difference between the series and shunt motor consists in the latter working with a wattless component of the armature current. The plain shunt motor works, therefore the better the lower the power factor. The series motor, in a way, is working with the useful component of the E.M.F., and its working ability improves with the power factor.

CHAPTER II.

THE SERIES COMMUTATOR MOTOR.

6. The Ideal Series Motor.

Following the principle put forward, we will now treat the ideal case first and then provide the ideal machine, with all the comparatively minor difficulties and complications. Our ideal series motor is a machine without saturation, without resistance in armature and field, without core loss and difficulties in commutation, and without self-induction in the armature. The diagram Fig. 26 is that of our machine.

The magnetic flux has hitherto been the only interconnecting factor between armature and field, as, apart from the leakage coefficient, armature and field flux were identical. Our series machine has a second important item common to the stationary and revolving parts—*i.e.*, the current. The formula for the useful power is

$$P = E \times I = \frac{1}{\sqrt{2}} \times s_a \times \frac{n}{60} \times I \times N_{\text{max.}} \times 10^{-8} \text{ watts,}$$

and for the wattless power consumed in the field

$$P_{Lf} = E_L \times I = \sqrt{2} \times \pi \times I \times z_f \times v \times N_{\text{max.}} \times c \times 10^{-8} \text{ volt-amperes.}$$

They have the common factor I. Therefore the ratio

$$k = \frac{P_{Lf}}{P} = \frac{E_L}{E} = 2\pi \times c \times \frac{z_f}{s_a} \times \frac{v}{n/60}, \quad . \quad . \quad . \quad (8)$$

i.e., *the ratio between the wattless power to produce the magnetic flux to the useful power is, apart from a constant, fixed by the ratio* $\dfrac{\textit{field turns}}{\textit{armature conductors}}$ *and the ratio* $\dfrac{\textit{frequency}}{\textit{speed}}$. The lower the frequency and the higher the speed, the smaller is the wattless power which must be spent to produce the field. With a certain motor with fixed s_a, z_f, and v, k is inversely proportional to the speed. Fig. 27 shows that the ratio k has a direct bearing upon the power factor—*i.e.*, the cosine of the angle ϕ, between the phases of the voltage e and current I. $\tan \phi = k = E_L/E = e_L/E$, or the power factor

$$\cos \phi = 1/\sqrt{1+k^2}.$$

If we wish to design a motor for a given frequency to run at a given speed, and if we further desire to obtain a certain $\cos \phi$, we can calculate the ratio of the field turns to the armature conductors by means of formula (8).

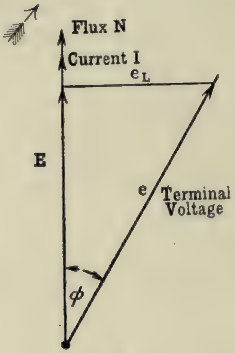


FIG. 27.

The latter may conveniently be written

$$k = \sqrt{\frac{1}{\cos^2 \phi} - 1}, \quad \text{or} \quad \frac{z_f}{s_a} = \frac{n/60}{v} \times \frac{1}{c} \times \frac{1}{2\pi} \times \sqrt{\frac{1}{\cos^2 \phi} - 1}$$

for a motor, for $n = 1{,}200$ revs. per min., $v = 25$ cycles per second, $c = 1{\cdot}12$, the following table is representative :—

$\cos \phi$	z_f/s_a
0·30	0·364
0·50	0·199
0·70	0·117
0·80	0·086
0·90	0·0554

It will be noted that a small ratio $\dfrac{\text{field-turns}}{\text{armature-cond.}}$ is a condition for a good power factor. It is the same as with induction motors. In both cases the field ampere-turns must be kept low, because the exciting energy must be supplied with alternating current, and is, therefore, expensive.

To obtain a power factor of 0·70 with our ideal two-pole motor the ratio of armature conductors to the field turns of a motor must be 0·117.

As the turns of the armature are one-half of the conductors and the conductors carry only one-half of the total current, we can say that, to obtain the power factor of 0·70, the field ampere-turns must not be more than 11·7 × 4 = 47 per cent. of the armature ampere-turns. The ratio $\frac{\text{field ampere-turns}}{\text{armature ampere-turns}}$ is familiar to us from the theory of direct-current machines and alternators, where it is of chief influence on the reactive drop of the voltage. With alternating current commutator motors it fixes the inductive drop and consequently the power factor. Through these elementary considerations such an important "constant" as the ratio of turns can be fixed. It is very easy to extend our study to multi-polar machines (say, 4 poles, Fig. 28), bearing in mind that our investigation refers to one magnetic circuit comprising a pair of poles. For the sake of convenience we give the machine a parallel wound armature

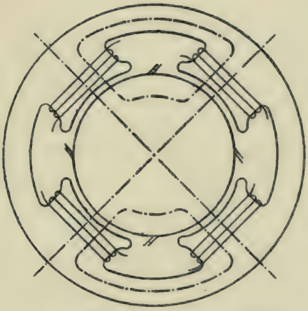


FIG. 28.—MULTIPOLAR MOTOR.

Giving s_a the same meaning as hitherto—*i.e.*, the total number of conductors on the circumference of the armature, we find

$$P = \frac{1}{\sqrt{2}} \times \frac{n}{60} \times s_a \times I \times N_{\text{max.}} \times 10^{-8} \text{ watts,}$$

the same as before. As the magnetic circuits have doubled, P_{Lf} is twice as great as before,

$$P_{Lf}=2\,.\,\sqrt{2}\times\pi\times I\times z_f\times v\times N_{\text{max.}}\times 10^{-8}\times c \text{ volt-amperes.}$$

z_f must be taken as the number of conductors per pair of poles. Putting $p/2$ = number of pairs of poles, we have generally

$$P_{Lf}=p/2\,.\,\sqrt{2}\,.\,\pi\times I\times z_f\times v\times N_{\text{max.}}\times 10^{-8}\times c \text{ volt-amperes.}$$

The ratio k is now

$$k=\frac{P_{Lf}}{P}=\frac{p}{2}\times 2\pi\times\frac{z_f}{s_a}\times\frac{v}{n/60},$$

and

$$\frac{z_f}{s_a}=\frac{p}{2}\times\frac{n/60}{v}\times\frac{1}{c}\times\frac{1}{2\pi}\times\sqrt{\frac{1}{\cos^2\phi}-1}.$$

A series armature equivalent to a parallel armature with s_a conductors would have $\frac{s_a}{p/2}$ conductors, and, therefore, with a series armature

$$k=2\pi\times\frac{z_f}{s_a}\times\frac{v}{n/60}\times c.$$

Here k becomes independent of the number of poles.

Introducing the ampere-turns instead of the turns or conductors, we find, generally, for any armature, series or parallel,

$$k=\pi\times c\times\frac{v}{n/60}\times\frac{\text{field ampere-turns per pair of poles}}{\text{total armature ampere-turns}}.\quad .\quad (9)$$

It is often convenient to compare the field ampere-turns per pair of poles with the armature ampere-turns per pair of poles; therefore,

$$\gamma=\frac{\text{field ampere-turns}}{\text{armature ampere-turns}}\text{ per pair of poles}$$

$$=\frac{1}{1{,}900}\times\frac{n}{p\,.\,v}\times\frac{1}{c}\sqrt{\frac{1}{\cos^2\phi}-1}.\quad .\quad (10)$$

The greater the number of poles, the higher the speed and the lower the frequency, the greater can be the ratio of $\frac{\textit{field ampere-turns}}{\textit{armature ampere-turns}}$ *to obtain a given power factor.*

A certain speed is specially significant, the synchronous speed—that is, the speed at which a synchronous motor with p poles would run.

For a four-polar motor, supplied with alternating current at 25 periods, the synchronous speed would be $n_s=750$ revs. per min.

Introducing n_s into the formula for γ we find

$$\gamma = \frac{2}{\pi} \times \frac{n}{n_s} \times \frac{1}{c} \sqrt{\frac{1}{\cos^2\phi} - 1}.$$

Introducing further the ratio $\beta = \frac{n}{n_s} = \frac{\text{actual speed}}{\text{synchronous speed}}$,

$$\gamma = \frac{2}{\pi} \times \beta \times \frac{1}{c} \times \sqrt{\frac{1}{\cos^2\phi} - 1},$$

$$k = \frac{E_L}{E} = \frac{\pi}{2} \times c \times \frac{\gamma}{\beta},$$

and

$$\cos\phi = \frac{1}{\sqrt{1 + \left(\frac{\pi}{2} \cdot c \cdot \frac{\gamma}{\beta}\right)^2}}.$$

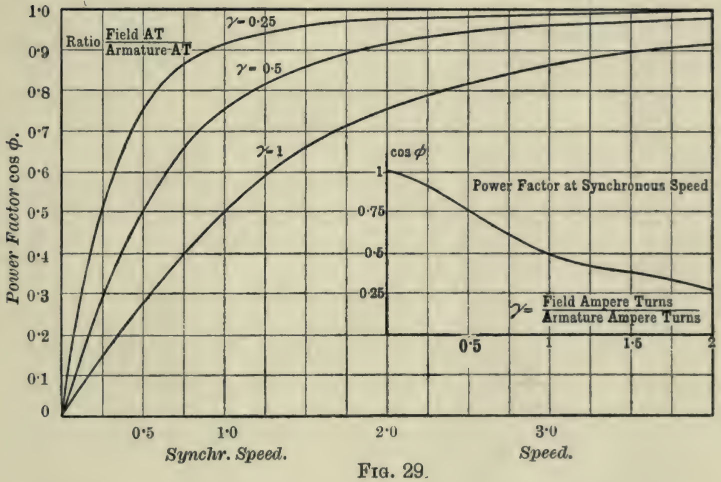


Fig. 29.

The three curves (Fig. 29) give the power factor as a function of the speed for the ratios $\gamma = \frac{\text{field ampere-turns}}{\text{armature ampere-turns}} = 0{\cdot}25, 0{\cdot}5,$ and 1. We have assumed c to be 1·1.

To obtain a power factor of 0·75 with the machine running at double the synchronous speed (say, four poles, 25 cycles, 1,500 revs. per min.), the ratio $\frac{\text{field ampere-turns}}{\text{armature ampere-turns}}$ must not exceed 1. With a direct-current machine γ is about 1, sometimes more, up to about 2, but very rarely less than 1.

If we were to design the alternating-current motor on similar principles, making $\gamma = 1$, a power factor of 0·85 would be obtained at about 2·90 × synchronous speed. Assuming we have to deal with a large railway motor where the speed may be fixed at, say, 725 revs. per min., the synchronous speed must be $725/2{\cdot}90 = 250$ revs. per min. With 25 periods the motor must have at least $120 \times 25/250 = 12$ poles; with $16\frac{2}{3}$ periods, 8 poles. If we keep the air-gap small and make $\gamma = 0{\cdot}5$, the speed need only be 2·2 × synchronous speed, and the number of poles can be reduced to 8 or 6 respectively. A very large number of poles is undesirable for many reasons; among others, it increases the number of commutator bars, makes the brushes come very close together, and reduces the interpolar space.

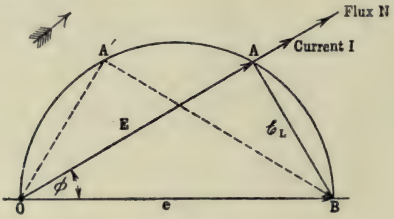


FIG. 30.

So we see that a low frequency, if possible below 25, is a condition for obtaining a good power factor. At the same time a very small air-gap, unknown with direct-current machines, is desirable—unless artificial means are employed to improve the power factor.

Hysteresis and eddy currents effect an indirect improvement of the power factor. Both these subjects will be dealt with in detail later.

To study the working conditions of the ideal motor at different loads we will build up a simple circle diagram (Fig. 30). In the diagram representing the relation between the three voltages on field, armature and at the terminals, the angle OAB between E and E_L is always 90 deg. Keeping the terminal voltage e constant, the point A at different loads moves on a circle with $OB = e$ as diameter. E_L is proportional to the current, provided the motor is not saturated, and therefore AB is representative of *the current* itself. If we have once determined the scale—1 ampere equal to a certain number of millimetres or inches—we can take the absolute value of the current as AB, the phase naturally remaining in the direction OA. *The useful power* is $P = e . I . \cos \phi$. As e is a constant, the watt component of the current $I \cos \phi$ is a measure of P. Drawing (Fig. 31) AC vertical on OB, AC is $I . \cos \phi$, and therefore can stand for P.

To obtain a measure of *the speed*, we remember that $OA = E$ is proportional to the current and the speed. E_L is proportional to the current only, and therefore the speed is equivalent to the ratio E/E_L, or OA/AB. A line in the diagram standing for this ratio is easily found. We draw OD vertical on OB and DF parallel to OB, then the triangle ODF is similar to OAB and $DF/OD = OA/AB$. As OD is a "constant" determined by convenience of scale only, DF is a measure of the speed.

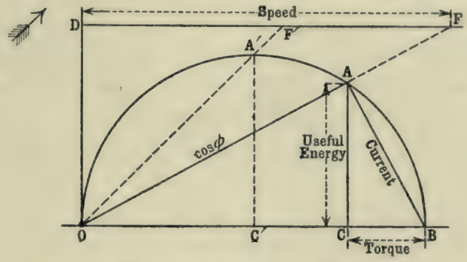


FIG. 31.—DIAGRAM OF IDEAL MOTOR.

It is convenient to have not only the angle ϕ, but also $\cos \phi$ given as a line. We can take OA for it, OB being put equal to unity.

It only remains to discover a line for *the torque*. The latter is proportional to the square of the current—*i.e.*, proportional to $(AB)^2$. As $AB/OB = BC/AB$; $AB^2 = BC \times OB$. OB as terminal voltage being constant, BC is a measure of the torque. OB itself would be *the starting torque* with full voltage on the terminal of the motor. It is interesting to note that

BC is also proportional to the wattless component of the current (I . sin ϕ), so that the torque is proportional to this wattless component, while the useful energy is proportional to the watt component of the current.

The preference for the mathematical or graphical representation is a matter of taste, but I think that many people will agree with me that a very simple diagram, as the present one, will lead more quickly to a clear result, and can more easily be borne in mind, than a more or less complicated formula.

From our diagram we see how at starting (speed DF equal to zero) F falls into D, cos ϕ being also zero, as the whole energy sent into the motor is the wattless energy consumption of the field, and the useful energy AC becomes zero. The current BA is then equal to BO, the highest value it can assume. As is customary, we will call this current taken by the motor at standstill under full voltage *the short-circuit current.*

The torque has the highest figure possible, CB becoming identical with OB. As the speed DF increases, the line OF swings round O towards OB. The torque BC gradually is reduced, reaching zero at infinite speed. The current likewise becomes smaller and smaller with increasing speed. Cos ϕ (OA) grows and approaches 1 asymtotically. The useful energy AC grows at first as the speed goes up, and reaches a maximum when A is on the top of the circle and the angle ϕ is 45 deg. ($\cos \phi = 0{\cdot}705$). Then the useful energy becomes smaller again as the current drops.

The linear diagram (Fig. 32) has been drawn by taking the different dimensions from Fig. 31. We have chosen the currents as abscissæ.

Prominent points in these diagrams are: (1) The point of starting; (2) the point where the maximum electrical energy is reached; (3) the normal load whatever it may be. We shall have to study these three conditions separately.

If we switch the full voltage on to a stationary motor, we produce the starting condition as represented in the circle diagram, when the point A falls into O. Our ideal machine has no drop in the armature, and therefore the latter does not consume any voltage at the speed zero, no E.M.F. being induced.

The whole voltage, therefore, is used up in the field winding. The current produced by this voltage is a good measure of the quality of the machine. If, for instance, at starting under full voltage the current produced is twice that current which the motor takes at normal load, then at normal current the voltage on the field is one-half of the full voltage. In this case the sine of the phase displacement of the current from the voltage at full load (Figs. 27 and 30) is $\sin\phi = E_L/e = \frac{1}{2}$, and consequently $\cos\phi = \sqrt{1 - \sin^2\phi} = 0{\cdot}868$.

So we see that an ideal motor, with a short-circuit current equal to twice the normal, has a power factor of 0·868. If, generally speaking, the short-circuit current is b times normal, then $\cos\phi_{\text{norm.}} = \sqrt{1 - 1/b^2}$.

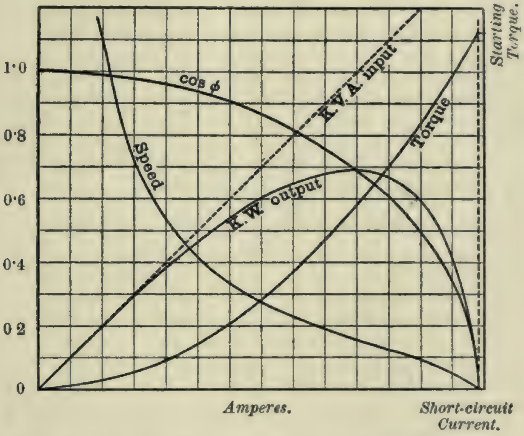


FIG. 32.—LINEAR DIAGRAM OF IDEAL SERIES MOTOR.

As the torque is directly proportional to the square of the current, *a machine with a short-circuit current equal to b times the normal has a starting torque equal to b^2 times the torque of the machine under normal conditions of load if under full voltage at starting.*

We must note that with increasing short-circuit current the torque grows with the second power, so that the starting torque per kilovolt-ampere input at starting is bigger the higher the short-circuit current of the machine. If means are provided to reduce the voltage at the terminals of the motor at starting we can for a given torque reduce the starting current further with a better motor.

The higher the short-circuit current the better the machine. A motor, where the field winding keeps the current down when it is stationary, will also suffer from the drop in the field under full-load working.

We can easily conceive that the short-circuit current i_K fixes the current scale of the diagram, $OB = i_K$. If we have calculated this figure, or obtained it from tests, we can draw the circle, and that fixes the current and power-factor curve of the whole machine. But not only this, it fixes also the maximum watt component of the current AC, which, as stated above, is equivalent to the electrical energy, the voltage being constant. The maximum AC is equal to the radius of the circle—*i.e.*, one-half of the short-circuit current—and, therefore, *the maximum overload capacity of an ideal series motor is equal to one-half of the energy consumed at short-circuit.*

A 500-volt machine, having, say, 100 amperes short-circuit current, can give out $\frac{1}{2} \times \frac{500 \times 100}{746} = 33\frac{1}{2}$ H.P. as a maximum.

We can ask, further, what is the maximum overload capacity of a 100 H.P. motor with a power factor of 0·85 ? The diagram gives the reply to this question. We draw a semi-circle, and AO under an angle, the cosine of which is 0·85. Then the radius of the circle, compared with the vertical AC on OB, gives the overload capacity. In our case the ratio of the maximum to the normal output is 1·27. The diagram tells us that if the power factor of the motor is given, the overload capacity can be calculated. We can see from Fig. 31 that

$$\text{Overload capacity} = \frac{0{\cdot}5}{\sin\phi \, . \cos\phi} = \frac{0{\cdot}5}{\cos\phi \, . \sqrt{1-\cos^2\phi}}.$$

Fig. 32, which is available for any motor, can be directly used to solve such questions.

It remains to fix the scale for the speed in our diagram. This can be done easily by considering (Fig. 31) the moment of maximum output (A on the top of the circle). Then OA is equal to AB and $E = E_L$; therefore, $k = E_L/E = 1$. Formulæ (8) and (9) furnish us with the result:

$$k = 1 = \frac{\pi}{2} \times c \times \frac{n_s}{n} \times \frac{\text{AT field}}{\text{AT armature}},$$

(AT = ampere-turns) and the speed DF′,

$$n = \frac{\pi}{2} \times c \times \frac{\text{AT field}}{\text{AT armature}} \times n_s.$$

Speed at maximum output = synchronous speed × $\frac{\pi}{2}$ × leakage coefficient × $\frac{\text{field ampere-turns}}{\text{armature ampere-turns}}$.

If the leakage coefficient is 1·1, the ratio $\frac{\text{field ampere-turns}}{\text{armature amp.-turns}}$ $= 0{\cdot}5$. DF′ is $= \frac{\pi}{2} \times 1{\cdot}1 \times 0{\cdot}75 \times$ synchronous speed $= 1{\cdot}30$ × synchronous speed.

This settles the scale for the speed. Naturally, if one speed for a certain output is known from calculation or test, the scale is fixed through this.

The study of the ideal series motor has given us a general idea of the working of the machine. It remains now to consider the complications and difficulties attached to the real motor.

7. The Self-Induction of the Armature.

The armature current produces a magnetic flux, the main portion of which passes through the pole shoe and twice through the air-gap. With a direct-current machine the only effect of this cross flux (Fig. 33) is the field distortion, with which we have connected the presence of the torque in the machine. With

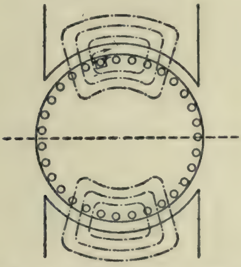


Fig. 33.—The Cross Flux.

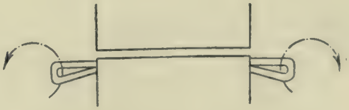


Fig. 34.—Leakage of End Connections.

an alternating-current motor this flux is an alternating one, inducing an E.M.F. in the conductors which it surrounds—in our case the armature conductors themselves. Besides, the flux passing through the pole shoe, some lines do not reach the latter at all. They close along the air space, or only across the slots. Others surround the end portion of the armature coils, having their paths entirely in the air (Fig. 34), or using partly the surrounding iron, shaft, end bells, bearings, &c. As a rule, about 85 to 90 per cent. of the cross flux passes through the pole shoe in the regular way, finding here the shortest path in the biggest section—provided the teeth are not very highly saturated by the main flux, which increases the reluctance of the path very much. Setting aside saturation for a moment, and taking the conductors as uniformly distributed over the circumference of the armature, then at a distance x (Fig. 35) from the centre of the pole the maximum flux density is

$$B_x = \frac{\sqrt{2} \cdot \pi}{10} \times \frac{I_a \cdot s_a}{\lambda} \times \frac{x}{t}.$$

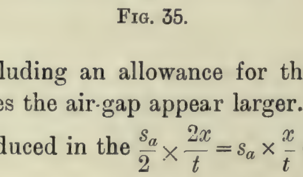


FIG. 35.

λ=air-gap, including an allowance for the opening of the slots, which makes the air-gap appear larger.

The E.M.F. induced in the $\frac{s_a}{2} \times \frac{2x}{t} = s_a \times \frac{x}{t}$ conductors by the flux passing at x is $E_x = \sqrt{2} \cdot \pi \times s_a \times \frac{x}{t} \times v \times B_x \times dx \times l \times 10^{-8}$ volts.

l=length of armature in pole in axial direction, and the total E.M.F. of self-induction in the armature

$$E_{La} = 2 \cdot \frac{\pi^2}{10} \cdot \frac{s^2_a}{t^2} \cdot \frac{I_a \cdot v}{\lambda} \cdot l \cdot \int_0^{\frac{b}{2}} x^2 \cdot dx \cdot 10^{-8}.$$

$$E_{La} = \frac{\pi^2}{60} \cdot 10^{-8} \times s^2_a \times I_a \times v \times (b/t)^2 \times \frac{b \cdot l}{2\lambda} \quad . \quad . \quad . \quad (11)$$

and the wattless power consumed in the armature is

$$P_{La} = \frac{\pi^2}{60} \cdot 10^{-8} \times (I_a \cdot s_a)^2 \times v \times (b/t)^2 \times \frac{b \cdot l}{2\lambda}.$$

The ratio pole width/pole pitch $= b/t$ is of the order 0·85 – 0·89 with this class of motor. The expression $2.\lambda/b.l$ is the reluctance of the air path for the main flux, which we called R.

Putting aside the magnetic resistance of the iron, we have the same reluctance R governing P_{La} as well as P_{Lf}.

$$P_{Lf} = 7{\cdot}8 \times \frac{(I_f \times z_f)^2}{R} \times v \times 10^{-8},$$

$$P_{La}/P_{Lf} = (1/47{\cdot}5) \times (I_a \cdot s_a/I_f \cdot z_f)^2 \times (b/t)^2.$$

Since in the series motor $I_a = I_f$,

$$P_{La}/P_{Lf} = (1/47{\cdot}5) \times (s_a/z_f)^2 \times (b/t)^2,$$

or, by introducing the ratio of ampere-turns,

$$\frac{P_{La}}{P_{Lf}} = 1/3 \times \left(\frac{\text{armature ampere-turns}}{\text{field ampere-turns}}\right)^2 \times \left(\frac{\text{pole width}}{\text{pole pitch}}\right)^2 \quad (12)$$

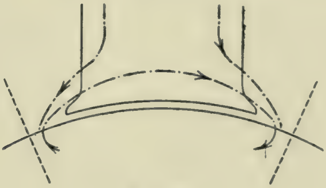


FIG. 36.

This ratio is independent of the number of poles. So we see that *we can express the wattless energy consumed in the armature as a percentage of the wattless energy consumed in the field.*

The armature ampere-turns may be 140 per cent. of the field ampere-turns, corresponding to a power factor of 0·79 when it revolves at twice synchronous speed. If, further, the pole width is 90 per cent. of the pole pitch, the inductive drop in the armature is 53 per cent. of that in the field winding. This ratio is not influenced by the fringe of the lines at the pole corners (Fig. 36), the spreading affecting field and armature flux in the same way. Also the reluctance of saturated teeth, which can only approximately be taken into account, affects both in the same direction, though not exactly in the same degree.

We may assume, when using formula 12, that the leakage lines of the field (coefficient *c*) practically compensate for the lines sent across slots and end connectors by the armature ampere-turns.

For practical purposes our formula 12 gives quite good results.

We can easily see that the influence of the armature self-induction is the same as if it were in the field, or as if the leakage coefficient *c* of the field was larger. If the ratio $\frac{\text{inductive drop in armature}}{\text{inductive drop in field}}$ is 0·53, as in the example just calculated, and the leakage coefficient is 1·1, all the formulæ we developed for the ideal motor, as well as the diagram, will hold good, provided we count $c = 1{\cdot}53 \times 1{\cdot}1 = 1{\cdot}69$.

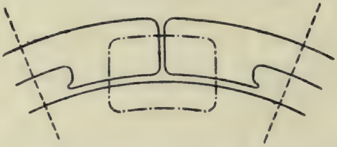


FIG. 37.—MOTOR WITH GAP IN MIDDLE OF POLE.

As the ratio of armature self-induction to field self-induction is fixed, it is evident that the former, just as the latter, is reduced proportionately if we increase the number of poles. Special designs have, however, been got out to reduce the self-induction of the armature. The simplest way for reducing the cross flux seems to be the introduction of an air-gap in the middle of a pole (Fig. 37). This almost halves the self-induction of the armature provided the gap is big enough.

More effectual is the use of a *compensating winding* embedded in slots on the face of the poles. This winding carries the main current, and is so dimensioned that it has practically the same ampere-turns as the armature itself, but acts in the opposite direction, and thus stops the cross flux to a very great extent (*see* Figs. 38 and 39). What cannot be stopped by the compensating ampere-turns is the magnetic leakage across the armature slots and the ends of the armature winding. In fact, this flux will be doubled, because the newly added compensating winding has its own leakage across the slots where it is embedded and round the coil ends. About 75 per cent. of the cross flux can be nullified by the method described.

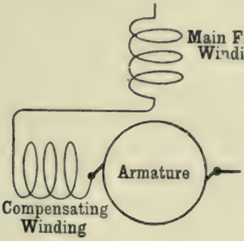


FIG. 38.—SERIES MOTOR WITH COMPENSATING WINDING.

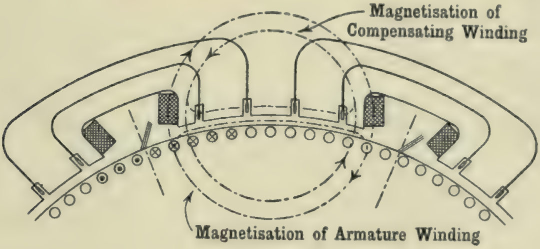

Magnetisation of Compensating Winding
Magnetisation of Armature Winding

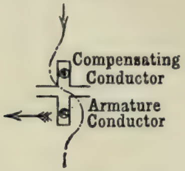
FIG. 39.

The armature inductive energy, which for $AT_a/AT_f = 1{\cdot}40$ amounted to 53 per cent. of the inductive field energy, would be reduced to about $0{\cdot}25 \times 53 = 13$ per cent. of the latter.

We see that the self-induction of the armature winding plus compensating winding is practically almost negligible when compared with the self-induction of the field winding proper.

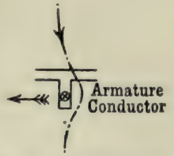
The method described last is very good, but it is rather expensive, as we have to provide a winding in the poles carrying as many ampere-turns as in the armature and containing more copper than the main field winding.

It may strike one that when the compensating winding is present the armature reaction cannot lift the curtain of lines to produce the torque, but this is only apparently so, as each line receives a sharp twist when passing into the armature through the sphere of action of the armature conductors. (Figs. 40 and 40A.)



Line of Force with Compensation.

Fig. 40.



Line of Force without Compensation.

Fig. 40A.

Instead of sending the main current through the compensating winding, use can be made of the fact that the cross flux is an alternating one. If the compensating winding is simply short-circuited in itself (Fig. 41) the cross flux produces a current in it which is opposing the armature current and reduces the cross flux itself. This means, naturally, is not quite so effective as the last method, as the compensating ampere-turns cannot be exactly equal to the armature ampere-turns. The latter must be so much bigger as to make a certain amount of cross flux pass for maintaining the current in the compensating winding. We may say that only about 50 or 60 per cent. of the cross flux can be compensated in this fashion. If this is considered sufficient, the method offers some advantage, as the number of turns is independent of the main current, and instead of a proper winding with several

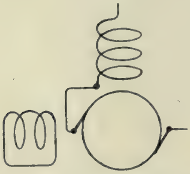


Fig. 41.—Compensating Winding Short-circuited.

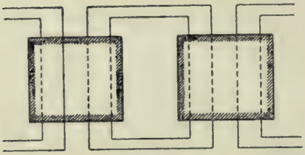


Fig. 42.

turns, each slot need only contain a solid copper rod connected to a similar one in the neighbouring pole. Fig. 42 shows this arrangement in diagrammatic form, when looking down at the face of the pole.

8. The Commutation.

With direct-current and alternating-current commutator motors the commutation takes place in the neutral zone—*i.e.*, when the two active sides of a coil (*a* and *b*) are in the interpolar space. The current passing from the brush into a commutator segment splits itself when entering the armature into two branches, the direction of the current in the coils of each branch being opposite to one another. Thus in Figs. 43 and 44 the current direction of the coils to the right of the brush is clockwise, that of the coil at the left-hand side inverse, so that the former coils under a north pole and the latter under a south pole produce a torque in the same direction. When the armature revolves counter clockwise the coils at the right-hand side pass to the left of the brush, and in passing the brush the current is reversed—*i.e.*, they undergo commutation. This process of commutation consists of short-circuiting the coil, the brush touching for a short time two commutator segments, and of releasing the short-circuit when the brush slips off a segment, at the same time embodying the coil into the winding branch carrying reversed current. With a direct-current machine the well-known difficulty arises that during the short time of the coil being under short-circuit the process of commutation has not been finished, and that a spark

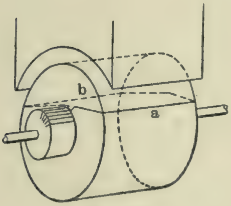


FIG. 43.

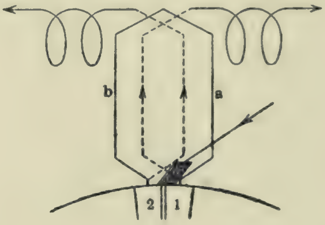


FIG. 44.

ensues when the coil is being opened. It is a well-known fact that we can speak of a "process of commutation," the current in one direction not being reduced to zero at the very moment of short-circuit, the current in the other direction not necessarily establishing itself at the moment of releasing the short-circuit. If we try to commutate too quickly the current lengthens the commutation by means of a spark. The reason for this reluctance to change is that every conductor carrying current produces a magnetic field which—if we make the current disappear—must disappear together with it and, in doing so, induces in the conductor itself an E.M.F. tending to keep the current flowing in the once established direction and at the old value. This E.M.F. of reactance is the greater the quicker the current alters. If, for instance, we let the current change its direction in infinitely short time the reactance would be infinite. The time of short-circuiting—*i.e.*, the time of commutation—is longer the smaller the number of commutator bars and the broader the brush in tangential direction. The time of commutation "required" is larger the higher the reactance of the short-circuited coil and the higher the current. Apparently we ought to be able to meet most requirements as to time of commutation if we could only make the brush thick enough, but if we increase the brush thickness beyond that of one segment it will short-circuit more than one coil at a time. If the brush were very broad indeed all coils in the interpolar space might be short-circuited, including a few right under the pole, having E.M.F.s induced in them. This would naturally lead to the production of current circulating in the coils and loading up the brush itself with idle current. So we see that the brush thickness is limited by the breadth of the interpolar space. On the other hand, if the brush is very thick several processes of commutation take place at the same time and influence one another unfavourably. Generally speaking, it is best to keep the brush thin, as long as the carrying capacity of the contact surface is sufficient for the current. Another means of checking the delay in commutation caused by the reactance is the use of hard carbon brushes, the contact resistance of which is very high. The latter would not only use up the energy of the reactance field becoming free in the short-circuit, it would also have a more or less governing influence on the distribution of the current when leaving the brush. Fig. 44, p. 39, shows two segments moving to the left under the brush. Through the movement the area of brush contact for segment 2 diminishes more and more, half the current which was passing straight from the brush to segment 2 gradually finding less resistance in passing to 1, flowing through the coil under commutation in the desired direction.

If we know the reactance of the coil we can take a fictive average reactance voltage equal to $\frac{\text{reactance field}}{\text{time of commutation}}$ as a measure for the tendency to spark. The reactance field is proportional to the current in the coil and inversely proportional to the magnetic reluctance. The latter is lower the greater the portion of the coil embedded in the iron and is also slightly influenced by the free coil ends. As the latter effect is generally small we can put

$$\text{Reactance field} = \text{constant} \times \text{core width} \times \text{current}.$$

The constant is dependent on the form of the slot where the coils lie, but does not vary very much with the slots now in general practical use. The time of commutation is smaller the higher the speed and the higher the number of commutator bars. So we obtain a formula for the reactance voltage

$e_R = \text{Constant} \times \text{revs. per min.} \times \text{commutator bars } m \times \text{current per coil } i \times (\text{turns per coil})^2 \times \text{core width}$; or

$e_R = \text{Constant} \times n \times m \times i \times (z_a/2m)^2 \times l$; $z_a/2m = w$.

It is not necessary to obtain this voltage in "volts," as we must bear in mind that the voltage does not actually exist, it being partly used up by producing a current. At any rate, the product

$$[n \times m \times i \times w^2] = C_K \quad . \quad . \quad . \quad . \quad . \quad (13)$$

affords a means of judging the sparking conditions of a machine, and actual experience has taught us the limits we have to keep for obtaining satisfactory commutation. The coefficient is about $2{\cdot}5$ to 3×10^8 for machines where the brushes must remain in the neutral position—*i.e.*, in such a position that the direction of rotation does not affect the sparking in any way. The constant $2{\cdot}5$ to 3×10^8 refers to direct current only. For alternating currents we should have to introduce the maximum of the current into the formula, or, introducing the root mean square of the current, to reduce the permissible value in the ratio $1 : \sqrt{2}$—*i.e.*, so that the constant becomes $1{\cdot}8$ to $2{\cdot}2 \times 10^8$. On account of other conditions (circulating current, as we shall see presently) which make the commutation worse, we ought to choose the constant not higher than 2×10^8.

Now, it is a well-known fact that the neutral position of the brushes with direct-current machines is not the most favourable. With a motor the commutation is better if the brushes have a backward lead. This has the effect of making the brushes follow the shifting backwards of the actual neutral zone, due to the armature reaction (section 1); it also lets the commutation take place when the short-circuited coil is under the trailing-pole corner. An E.M.F. is then induced in the coil during short-circuit, which has the direction of the counter E.M.F. of the armature winding (opposite to that of the current in Fig. 44), and therefore is reversing the current in the coil under commutation. In a reversible motor, brush shifting from the neutral is almost excluded, and cannot be easily applied to alternating-current motors, which are, as a rule, required to be reversible. On the other hand, if we shift the brushes backward the armature ampere-turns not only exercise a cross-magnetising, but also a demagnetising, influence. They would counteract the field ampere-turns, and therefore a larger number of the latter would be required to create a certain flux. This is a more serious matter with alternating-current machines than with direct-current ones, as the armature ampere-turns are far greater in proportion to the field ampere-turns than in the latter. A glance at Fig. 45 tells us

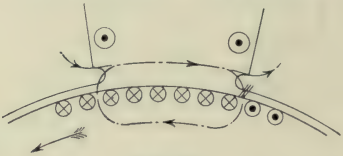


FIG. 45.

that the leakage of the field will be considerably increased, and, therefore, the power factor would suffer. If, for instance, the ratio of field ampere-turns to armature ampere-turns were 0·6, a brush shifting to such an extent that the commutated coil is under the pole corner would involve 30 to 40 per cent. more field ampere-turns, the span of the armature coils being assumed equal to full pole pitch. With a non-compensated machine, brush shifting would make little improvement in sparking, because the comparatively high armature ampere-turns, by their strong demagnetising power at the exact place in the field where the commutation goes on, would actually reverse the direction of the field at this place, so that we should not obtain the expected benefit from the brush shifting. Further, it is clear that, in actual practice, a shifting of brushes together with the compensating winding would be a matter of great difficulty. These are the reasons why brush shifting is out of the question with a plain series motor.

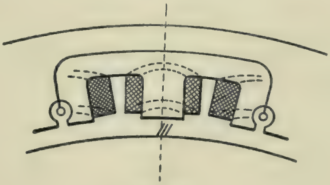


FIG. 46.

With direct-current machines auxiliary poles are in use, which are placed between the main poles and are excited by the main current in series with the armature winding (Fig. 46) Their effect is similar to that of the trailing pole-tip under which the brushes are shifted, in order to facilitate commutation by means of the flux of the main pole. The auxiliary pole produces such a flux, but with the advantages (1) that the brushes can remain in the neutral position, the flux being now shifted to the brushes, whereby reversibility is ensured; (2) that the commutating flux increases proportionately as the current increases. Actually, the winding on the commutating poles it nothing but a continuation of the compensating winding, except that it contains a few more turns than the remaining single coils, so that many firms are providing for this pole with an alternating-current commutator motor.

It is very easy to apply formula (13) for the reactance voltage if all the required data are given, but for application in a more general way we will put it into a different form, combining it with the formula for the E.M.F. of the armature.

Taking a series-wound armature as basis, we have

$$\mathrm{E} = s_a \times \frac{p}{2} \times \frac{n}{60} \times \mathrm{N} \times 10^{-8} \text{ volts.}$$

Combined with formula (13) we find, after some transformation,

Total flux from all poles together

$$p \times \mathrm{N}_{\mathrm{max.}} = 1{\cdot}12 \times 10^{18} \times \text{horse-power} \times w \times l. \quad . \quad (14)$$

Formula (14) is in words :—

The minimum total flux a machine must work with so as to run sparklessly is proportional to the horse-power, the number of turns per section, and to the width of the armature.

With larger machines the number of turns per section w is 1, so for these we can put

Flux per centimetre core width

$$= p \,.\, \mathrm{N}_{\mathrm{max.}}/l = 1{\cdot}12 \times 10^{18} \times \text{horse-power.} \quad . \quad . \quad (15)$$

Taking the commutator diameter as large as possible, so as to reduce the number of turns per section to a minimum, say, commutator diameter = 0·85 × armature diameter, and applying as high a number of commutator bars as practicable, we can put

$$p \times \mathrm{N} = \text{constant} \times \sqrt{\frac{\text{H.P.}}{\text{revs.}}} \times \sqrt{\text{volts}} \times \sqrt{\frac{\text{length of armature}}{\text{diameter of armature}}}.$$

In this general form the formula will do good service in forming a general idea of the amount of flux required without our knowing anything but horse-power, speed, voltage and approximately the ratio of length to diameter of armature, a figure varying between 0·5 and 0·9, the square root of this figure being 0·7 to 0·95.

9. The Circulating Currents in the Short-circuited Coil.

Looking at Fig. 47 we notice that the coil, short-circuited by the brush, though entirely out of the range of the poles, is under the inductive influence of the whole of the flux issuing from the pole. The coil under short-circuit actually surrounds the flux entering the armature, and in our case this flux being an alternating one, induces an E.M.F., and, consequently, a current in this coil. If the short-circuit was perfect and the resistance and self-induction of the coil and the resistance of the brush contact very small, this "circulating current" would be extremely heavy. It would not only be sufficient to cause considerable ohmic loss, but would also load up the contact surface of the brush, in most cases up to the glowing point; and, besides, its demagnetising action on the field itself would be seriously felt. If there was practically no leakage and resistance, the current in the single armature coil would assume such a value that it would almost neutralise the action of the field coils and prevent any flux from passing through the armature. The amount of current flowing in the coil might, under these conditions, be more than *ten* times the current it is supposed to carry ordinarily. The effect on the brushes would not only be glowing but also sparking, due to the circuit being broken. We see, therefore, that this current must be checked. It must be reduced to a reasonable amount, say less than normal current, to prevent any of its bad effects. The most natural way to do this is the introduction of ohmic resistance in the circuit formed by the brush, commutator

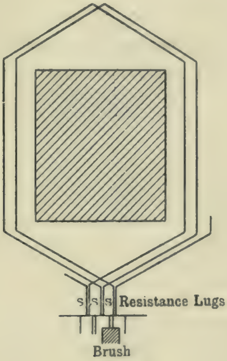


FIG. 47.

segments, and the connection between both—*i.e.*, the commutator lugs or connectors, as they are generally called (Figs. 47, 47A and 48). These connections are generally used as a resistance, either by having a great length or by being made out of a metal of low conductivity. It would be undesirable to give the coils or the brush a higher resistance than we can help, as these parts have to carry continually the main current. The lugs are particularly suited for serving as resistance, as they have to carry the main current only for a very short time, when the brushes are in contact with their commutator segments; otherwise they are idle. Naturally, even in this favourable case, the reduction of the circulating current must be bought by a certain amount of loss, for a high resistance giving a very small circulating current indeed will also cause quite an appreciable loss, due to its carrying the main current. A simple calculation will give us an idea of these relations. We will call the voltage induced in the coil under commutation by the alternations of the main field ϵ, the resistance of each connector ς, and assume that ς is so big that the resistance of the main coil is practically negligible compared with it. In ς shall be included the average contact resistance the current finds when passing from the brush to the segment, due account being taken—as far as possible—of the contact resistance being smaller the higher the current density.

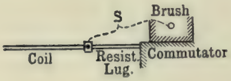


FIG. 47A.

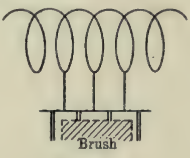


FIG. 48.

It is evident that the self-induction of the coil which might reduce the current and cause phase displacement is comparatively small, the current of the coil being more than 10 times the normal* if ς is zero. We will assume that the brush had

---

* Thinking of an alternator with a short-circuit current equal to 10 times the normal current, one will understand this statement.

the exact width of one segment (Fig. 49), thus short-circuiting only one coil at a time. The circulating current would then be $i_c = \epsilon/2s$, and would cause a loss, $p_c = \epsilon^2/2s$, in the lugs under one brush, or $p_c = \epsilon^2/s$ for a pair of (plus and minus) brushes. The main current I flowing through the lugs in parallel causes the loss $p_m = I^2 \times \frac{s}{2} \times 2 = I^2 \times s$ per set of brushes. Therefore, the total loss in the lugs through circulating and main current

$$p_c + p_m = \frac{\epsilon^2}{s} + I^2 \times s.$$

A simple differentiation shows that the total loss is the smallest if p_c and p_m are equal—*i.e.*, if $\epsilon^2/s = I^2 \,.\, s$; $s_{\text{min.}} = \epsilon/I$, and the circulating current i_c equal to the main current.

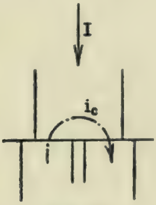


FIG. 49.

If the brush width is equal to that of two commutator segments, it short-circuits mostly two coils at a time ; then

$$p_c = 4 \,.\, \frac{\epsilon^2}{s} \text{ per pair of brushes,}$$

$$p_m = \tfrac{2}{3} \,.\, I^2 \,.\, s \text{ per pair of brushes.}$$

The minimum of the total loss is reached if $p_c = p_m$ or $4 \,.\, \frac{\epsilon^2}{s} = \frac{2}{3} \,.\, I^2 \,.\, s$; that is,

$$s_{\text{min.}} = 2{\cdot}45 \,.\, \frac{\epsilon}{I},$$

$$(p_c + p_m)_{\text{min.}} = 3{\cdot}25 \times \epsilon \times I ; \quad i_{\text{circ.}} = 0{\cdot}41 \times I.$$

For a brush, short-circuiting x coils, we find

$$p_c = x^2 \,.\, \epsilon^2/s ;$$

$$p_m = 2 \times \frac{I^2 \times s}{x + 1} ;$$

and the condition for the minimum of loss

$$s_{\text{min.}} = \frac{\epsilon}{I} \times x \sqrt{\frac{x+1}{2}},$$

$$(p_c + p_m)_{\text{min.}} = \epsilon \times I \times \frac{2{\cdot}82}{\sqrt{\frac{1}{x} + \frac{1}{x^2}}} ; \quad i_{\text{circ.}} = \frac{I}{\sqrt{2(x+1)}}.$$

The greater the number of segments covered by the brush the greater the losses in the lugs, the higher the resistance of the lugs must be and the smaller the circulating current compared with the main current.

These calculations are naturally only approximate, the distribution of the main current over the segments being made non-uniform by the commutation process itself, the current tending to pass chiefly through the segment about to leave the brush. But the high lug resistance itself counteracts this tendency and makes our assumption of uniform distribution more correct.

We have found in a general way the condition for making the losses in the lugs as small as possible. Naturally this minimum does not entirely settle the amount of lug and brush resistance; other considerations come in as well—for instance, mechanical ones and the current density in the brush. The latter will be satisfactory since, with the brush covering three or two segments, the circulating current is only 35 to 41 per cent. of the main current, if we comply with the condition for the minimum losses. We can, however, make the resistance somewhat larger or smaller, as the losses up to a certain point change only very slowly provided that we do not depart too far from $\varsigma_{\text{min.}}$ If $\varsigma = 1{\cdot}75 \times \varsigma_{\text{min.}}$ the circulating current is only about 20 to 23 per cent. of the main current, and the losses are only 15 per cent. more than the theoretical minimum. This will be seen from Fig. 50, representing the losses and the ratio $\dfrac{\text{circulating current}}{\text{main current}}$ for different lug resistances, the brushes being taken as covering two segments.

We can easily form an idea of the loss in the resistance lugs expressed as a percentage of the useful electrical energy, the latter being $P = E \times I$. With a parallel armature and two commutator segments covered by the brush

$$(p_c + p_m)_{\text{min.}} = 3{\cdot}25 \times \epsilon \times I = 3{\cdot}25 \times P \times \frac{\epsilon}{E}.$$

This formula holds good whatever the number of poles and number of sets of brushes, as, by introducing I, the total current, the number of brush sets has been taken account of.

To find the ratio between ϵ and the E.M.F. E of the armature we put (*see* formula 2)

$$E=\frac{1}{\sqrt{2}}\times s_a\times\frac{n}{60}\times N_{\text{max.}}\times 10^{-8}.$$

m = number of commutator segments,
$s_a/2m$ = number of turns per coil.

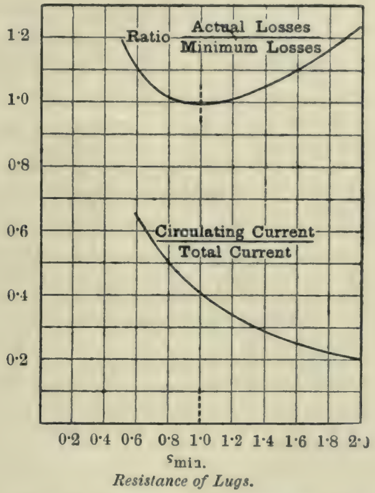


FIG. 50.—LOSSES IN RESISTANCE LUGS.

The E.M.F. induced by the main flux into the $s_a/2m$ turns of an armature coil is

$$\epsilon=4{\cdot}44\times\frac{s_a}{2m}\times v\times N_{\text{max.}}\times 10^{-8}$$

$$=4{\cdot}44\times\frac{s_a}{2m}\times\frac{n_s}{120}\times p\times N_{\text{max.}}\times 10^{-8},$$

and consequently

$$\frac{\epsilon}{E}=\frac{\pi}{2}\times\frac{p}{m}\times\frac{n_s}{n},$$

or, if the brushes cover two segments,

$$\frac{\text{Loss in resistance lugs}}{\text{Useful energy}}=\frac{5}{m}\times\frac{n_s}{n}\times p$$

$$=\frac{5\times\text{number of poles}}{\text{number of commutator segments}}\times\frac{\text{synchronous speed}}{\text{normal speed}}.\quad(16)$$

This formula enables us to estimate the losses in the resistance lugs and brushes, if only speed, number of poles, frequency and number of segments are given. A motor for 15 cycles, four poles (synchronous speed 300 revs. per min.), 750 revolutions and 400 commutator segments has a minimum loss in the resistance lugs and brushes equal to $\frac{30}{400} \times \frac{300}{750} \times 100 = 3$ per cent. of the useful energy of the machine. The condition for the minimum loss was equality of losses due to equalising current and main current. The main current, therefore, must cause 1½ per cent. loss ; in other words, the drop in voltage over brush and resistance must be about 1½ per cent. of the voltage in the armature. If the latter is 200 volts, and we have selected brushes giving 2×1 volts $=2$ volts over the brushes, the drop over the resistance ought to be $\left[\frac{1{\cdot}5}{100} \times 200\right] - 2 = 1$ volt. Allowing for the resistance being higher than required for the minimum, the total loss will go up to about 3½ per cent. and the drop in the lugs alone to about 2 volts. If the machine has approximately 80 per cent. efficiency, the losses in the lugs will affect the efficiency $3\frac{1}{2} \times 0{\cdot}80 = 2\frac{3}{4}$ per cent. With 200 commutator segments (smaller machines) the loss would be double.

A fact which we can derive from this example is that the resistance in the brush contact is very nearly sufficient to keep the circulating current down to a moderate amount. In fact, if the voltage of the motor is kept low enough, and a carbon brush is selected having high contact resistance, the resistance lugs can be dispensed with altogether. Though through this expedient the size of the commutator becomes rather large, it has been applied in actual practice with quite large machines for traction work. It is certainly desirable not to use resistance lugs as far as the starting of the motor is concerned. The resistance lugs are being designed for intermittent use at full load when they have to carry current only as long as the brushes touch their respective segments. Therefore, there is some danger of their burning out when the motor is stationary or moving slowly.

A given machine, say, for 25 cycles, having its resistance lugs adjusted to give the minimum loss has, as we found, a circulating current approximately equal to the normal current. If we were to increase the frequency to 50, the circulating current would double $\Big(\frac{n_{\text{synchr.}}}{n_{\text{normal}}}$ or ϵ double$\Big)$ and the machine would spark. The resistance ought to have a different value for each frequency. In the example we just calculated for 15 periods the minimum loss in the resistance would be 6 to 7 per cent. at 30 periods instead of 3 to 3½ per cent. at 15. *High frequency, therefore, affects the efficiency unfavourably as well as the power factor.*

For approximate estimating it is desirable to eliminate construction details, in our case the number of commutator segments, out of formula (16).

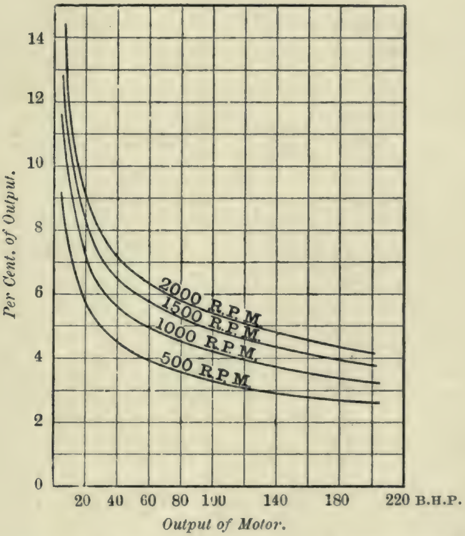


Fig. 51.—Losses in Lugs and Carbons at Synchronous Speed.

This can be done by going into the design in a general way, determining the armature diameter from the output, and assuming the commutator diameter to be as large, and the number of commutator segments as high, as practicable.

Plotting the loss in the lugs and brushes for the ratio $\frac{\text{synchronous speed}}{\text{running speed}} = 1$, we obtain approximate curves (Fig. 51), giving the loss in the lugs as the percentage of the output. These figures are to be multiplied by the ratio $\frac{\text{synchronous speed}}{\text{normal speed}}$.

Our formulæ, developed for machines with parallel armatures, must be slightly modified and correctly interpreted for series armatures. A simple comparison will clear up this question.

We shall take the brush width equal to that of one segment only. Then, with a parallel armature,

$$p_c = p \cdot \frac{\epsilon^2}{2s}; \qquad p_m = 2 \cdot I^2 \cdot \frac{s}{p};$$

$$(p_c + p_m)_{\text{min.}} = 2 \cdot \epsilon \times I; \qquad s_{\text{min.}} = \frac{\epsilon}{I} \times \frac{p}{2}.$$

With series armatures we can reduce the number of brush sets down to a single pair. In this case we have

$$p_c = \frac{p^2}{4} \cdot \frac{\epsilon'^2}{s}; \qquad p_m = I^2 \cdot s.$$

$$(p_c + p_m)_{\text{min.}} = I \times \epsilon' \times p; \qquad s_{\text{min.}} = \frac{p}{2} \times \frac{\epsilon'}{I}.$$

Apparently the loss in the lugs of a series armature cannot be kept so small as with a parallel one, but bearing in mind that with a parallel armature we have $p/2$ times the number of conductors than in the equivalent series armature, and therefore if the motor is otherwise the same $\epsilon' = 2 \cdot \epsilon/p$, and for series armatures with two brush sets and parallel armatures alike

$$(p_c + p_m)_{\text{min.}} = 2 \times I \times \epsilon.$$

The ratio $\frac{\epsilon'}{E}$ for series armatures becomes

$\frac{\epsilon'}{E} = \frac{\pi}{m} \times \frac{n_s}{n}$; and for brush width = two segments :

$$\frac{\text{loss in resistance lugs}}{\text{useful energy}} = \frac{5}{m} \times \frac{n_s}{n} \times p.$$

Making the number of brushes equal to the number of poles with the series armature, we have

$$p_c = p \cdot \frac{\left(\frac{p}{2} \times \epsilon'\right)^2}{2s}; \qquad p_m = 2 \cdot I^2 \frac{s}{p};$$

$$(p_c + p_m)_{\text{min.}} = I \times \epsilon' \times p; \qquad s_{\text{min.}} = \frac{\epsilon'}{I} \times \frac{p^2}{4}.$$

When developing these formulæ for the series armature we assumed that there was some resistance or inductance between the brushes of equal polarity so that only a small current could flow from one to the other. If this is not the case the losses will be higher, with four and six polar machines about 25 to 30 per cent.

If the resistance is not altered—*i.e.*, increased with the number of the brushes—and it has once been adjusted to give the minimum losses with two brush sets, then the putting down of more brushes increases the losses in the resistance connectors.

To show how the character of the current in the coil under commutation is affected by the circulating current, we draw an ordinary commutation curve for direct current (Fig. 52). Up to the moment when the coil is short-circuited the current has the value $+I'$; when it is leaving the short-circuit the current is reversed to $-I''$.

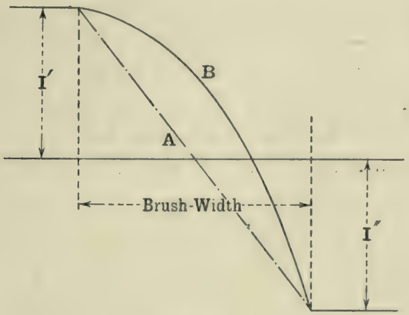


Fig. 52.

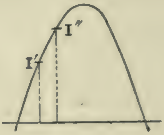


Fig. 53.

Without the effect of self-induction the change would follow a linear law (A); through the delaying influence of the reactance, however, the current will be deformed into B. The time of commutation being very short generally, compared with the time of one period of the alternating current, I'' is practically equal to I' (Fig. 53). It is actually slightly larger for the rising half of a half-period and slightly smaller for the dropping half. The circulating current, being due to a voltage induced by the main flux, is zero when the main current is a maximum (2, Fig. 54), and, therefore, does not interfere with the commutation of the maximum of this current. It is opposing the initial armature current when the latter rises (1 – 2) and helping the initial armature current when it falls (2 – 3). In Fig. 55, drawn for the period 1 – 2, B is the original commutation curve, C the circulating current, and D the total current resulting from both. Fig. 56 shows the corresponding curve for the period 2 – 3. We see that during the falling of the current the commutation is more difficult than during rising; in the latter case the circulating current flows in the direction of the current which we wish to produce in the short-circuited coil. The correctness of our investigation, especially the relative direction of the currents, can be easily verified by

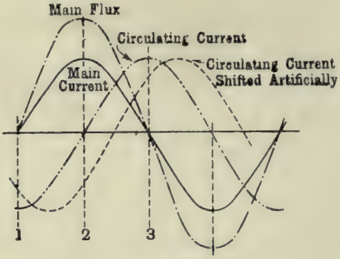


FIG. 54.

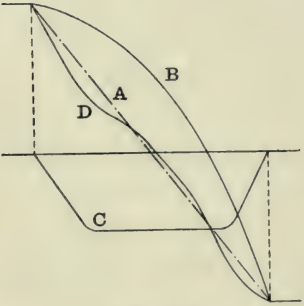


FIG. 55.

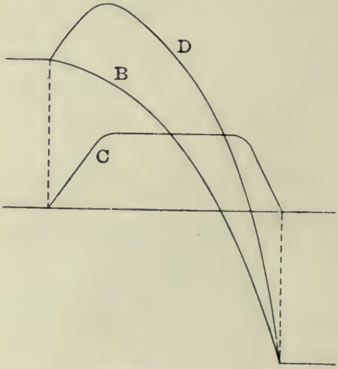


FIG. 56.

looking at Fig. 57, direction of rotation and current having been found by applying the curtain rule. We have to remember that the circulating current magnetises when the flux tries to disappear (direction of $i_{\text{circ.}}$ being the same as that of i_f).

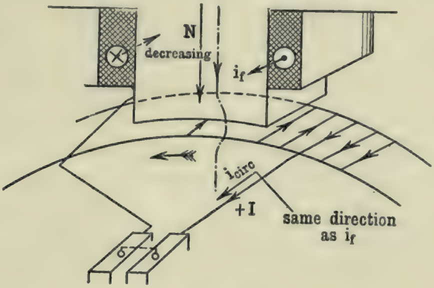


FIG. 57.

The fact that during the period 1—2 the circulating current tends to improve the commutation prompts us to artificially lengthen this period and thus make the otherwise harmful circulating currents do useful service. The circulating current must for this purpose have a component in phase with, but opposite to, the armature current. The circulating current is thus shifted into the dotted position in Fig. 54. This can be achieved by placing an inductive branch (choking coil) across

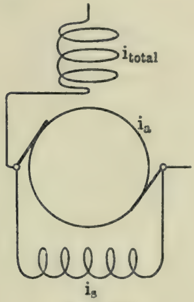


FIG. 58.

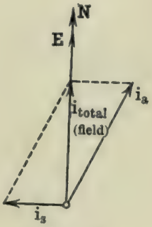


FIG. 59.

the armature terminals. This causes a current, i_s, to flow in the field besides the armature current, thus shifting the field current, and with it the main flux and the circulating current, in the desired direction (Figs. 58 and 59).

The same object is attained by placing a non-inductive resistance in parallel with the field winding, in this case the armature current composed of i_f and i_r being shifted as intended (Figs. 60 and 61).

It is evident that the method is not perfect, the bulk of the circulating current remaining in quarter-phase with the main current, thus being partly beneficial, partly harmful. On the other hand, the form of curve B depends on the time of commutation—*i.e.*, on the speed—whilst C is almost independent of it, and therefore the commutation can be made good to some extent at a certain speed only. The advantage obtained must be bought by either making the machine worse through the self-induction in parallel with the armature or by the reduction in efficiency through the watts consumed in the resistance across the field terminals, though this amounts to only about 2 per cent.

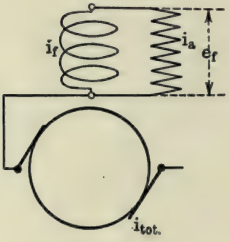


FIG. 60.

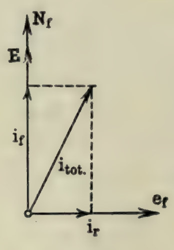


FIG. 61.

There is one item that helps the effect of both these expedients—*i.e.*, the hysteresis, which makes the main flux lag somewhat behind the exciting current, and in this way shifts the phase of the circulating current in the right direction. The hysteresis lag amounts to 10 deg. or 15 deg. Also the self-induction of the short-circuited coil—which is, however, as pointed out, only small compared with the resistance, the latter being large to check the absolute value of the circulating current—tends to shift the phase of $i_{\text{circ.}}$ so that it helps commutation. We shall have to deal with this subject again when considering the Winter-Eichberg motor and others.

Another method of checking the circulating current, besides that of using resistance, is to interrupt the circuit. There are two distinct but identical armature windings without any electrical connection, every commutator segment of one winding being sandwiched between two of the others (Fig. 62). If, futhermore, the brush is not broader than one segment it cannot short-circuit a coil. The current branches from the brush into the two windings in parallel. With this method the great trouble arises that an equalising current may flow from one winding over the brushes into the other winding. It is practically impossible to arrange on a core two windings so that there is absolutely no difference of electrical potential between them. If the latter exists, and we connect the windings by the brushes, heavy currents pass (the windings being nearly bifilar and having small self-induction) and sparking and pitting of the commutator ensues. This method has, therefore, found practically no application.

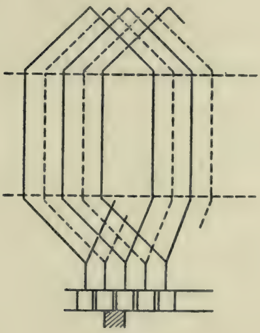


Fig. 62.

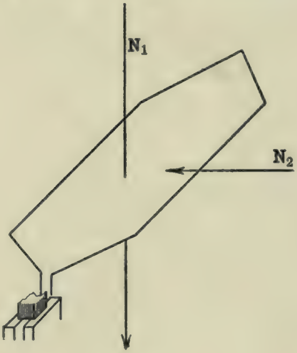


Fig. 63.

There are altogether three ways of checking an undesirable current: (1) Introducing a resistance, inductive or non-inductive. The former, however, is out of the question in our case on account of the sparking caused by reactance. (2) Interrupting the circuit. Both these have been dealt with. (3) Introducing an E.M.F. of such a value that the E.M.F., tending to produce the undesirable current, is counterbalanced.

The latter method would be ideal, but to induce an E.M.F. equal to that induced by the main flux would mean applying a flux of the same magnitude, and this is impossible, because we should not get any torque with two equal and opposite fluxes. There is, however, one solution to this question. We can put an auxiliary flux, N_2, at right angles to the main flux N_1, so that the conductors in the short-circuited coil cut the lines of N_2 when moving (Fig. 63). The E.M.F. induced in this way can be made equal and opposite to that producing the circulating current under certain conditions. It will be easily realised that one of these conditions is a phase displacement of 90 deg. between N_1 and N_2, N_1 inducing by change of intensity, its effect being zero when it passes over its maximum, N_2 inducing by its lines being cut, and, therefore, its effect being at its height when N_2 has reached the highest value. There can exist only one speed at which the E.M.F. induced by cutting N_2 is equal to that induced by the fluctuation of N at a certain fixed frequency. The conditions are fulfilled in the Winter-Eichberg-Latour machine, which we shall deal with in a special chapter, but also with the plain series motor an auxiliary flux, N_2, in the neutral zone has been made use of for damping the circulating currents, at least at one speed. A special winding, W_s (Fig. 64), has been placed on the commutating pole besides the ordinary series commutating coil (W_k), W_s being excited from the main voltage or a voltage proportional to it. Sometimes the armature, or some mixed voltage, is used for excitation in order to correct the phase of the flux N_2 produced by W_s. If the power factor were unity, an excitation from the total voltage would be correct, because then there would be 90 deg. phase displacement, as required, between N_2 and N_1, the latter being created by the main current. As the power factor is actually smaller than 1, W_s might with advantage be excited from the armature terminals only, their voltage being more nearly in phase with the current. Some provision has to be made to counteract the mutual induction between W_s and W_k, either by inserting a choking coil in W_s or by placing W_s and W_k in different slots (Fig. 65), or some similar arrangement.

A similar effect can be attained by placing a resistance, R, in parallel with the compensating winding (Fig. 66), which gives the current in this winding a component, i_s (Fig. 67), shifted 90 deg. against i, and producing a flux component, N_2, in the desired phase. The energy consumption is not very large, say, about 2 per cent. only. Instead of a resistance in parallel with W_k a choking coil in parallel with the armature may be used.

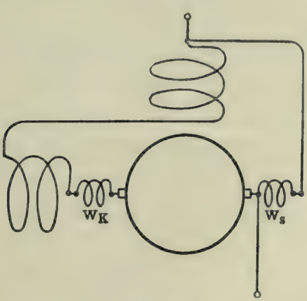


FIG. 64.

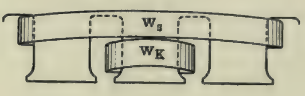


FIG. 65.

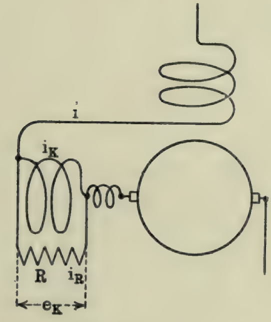


FIG. 66.

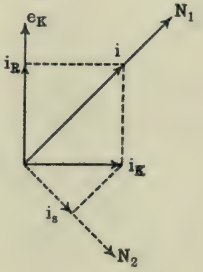


FIG. 67.

10. The Core Loss.

It is not our intention to go into the very complex question of the calculation of core loss generally, to study the influence of eddy currents, its dependency on filing the slots of the machine and other causes, which tend to increase the core losses in any machine, alternator, induction motor or direct-current generator. We take it to be a well-known fact that we cannot obtain exact results if we are to calculate the core loss, using a hysteresis curve, determined experimentally from a sample of the iron. The actual losses come out to be double, and even three or four times, as much as found by simply multiplying the volume of the iron by the loss per unit at the working density taken from the above-mentioned curve. It is only the special behaviour of the alternating-current motor with regard to core loss we wish to point out. In the stationary part the flux changes at the well-known periodicity and the losses can be calculated just as for the stator of an induction motor. There are losses in the pole face due to the armature reaction sending the flux through it and increasing the density at certain places, at others weakening it. If the machine is compensated, this flux from the armature will disappear, but in place of it quite an appreciable leakage flux around the slots, which carry the compensating winding, will establish itself and increase the losses. If the slots of the armature are very wide compared with the air-gap, hysteresis losses and a small amount of eddy currents will be produced in the pole face, due to the movement of the armature teeth from which the flux issues. This loss affects only a small zone where the flux is not uniform; it penetrates the pole to an extent somewhere near a quarter of the slot width (Fig. 68).

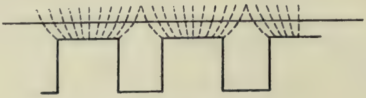
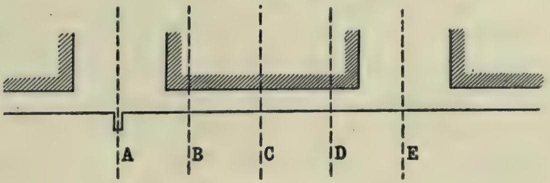


Fig. 68.

The peculiar character of the hysteresis in the armature is caused by the flux having an alternating character to start with, and at the same time the iron revolving relative to it. The hysteresis loss, say in a tooth of the armature, is dependent only on the number of cycles of magnetisation it passes through and on the maximum density obtained. The difficulty lies in the fact that neither one nor the other is strictly defined. Supposing a motor runs at half its synchronous speed, and consider a certain tooth which is exactly in the neutral position (Fig. 69) between two poles at the moment when the flux is zero; whilst the flux is rising to its maximum value the tooth moves a quarter of the pole pitch. During the following quarter of the cycle the flux falls to zero whilst the tooth comes to the centre of the pole, &c., as indicated in curve I, Figs. 70 and 71. We see that the tooth in question would be

Fig. 69.



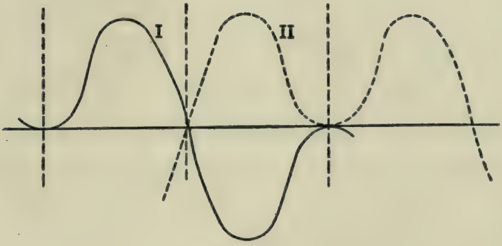


Fig. 70.

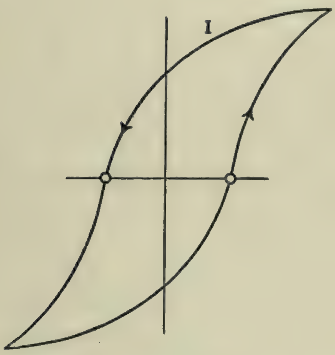


Fig. 71.

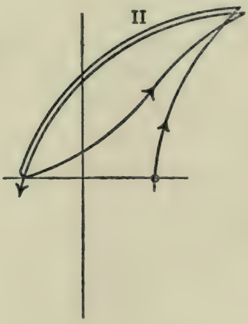


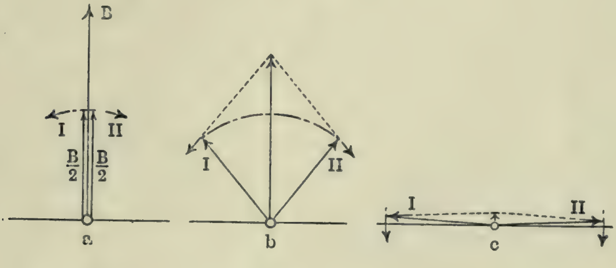
Fig. 72.

magnetised and demagnetised at the same frequency as the current. Another tooth, say one which is in the centre of the pole at the moment when the flux is zero, will be in the neutral position when the flux is zero again. Then the magnetisation changes its sign, whilst the tooth is moving into the sphere of the following pole. Through this double change the tooth receives again a magnetisation in the same direction, so that two half-loops of hysteresis are formed instead of a complete loop, as with the tooth considered first (curve II, Figs. 70 and 72). If we were to investigate the process of magnetisation for different points of the teeth, or of the armature core proper, we should find that the character of the cycles of magnetisation which the different parts of the core have to undergo differ vastly from one another. They change with the speed, and the law which the losses follow being different for hysteresis and eddy currents is not simple to determine, as is apparent from this brief investigation. To come to a practical result we shall make use of a method largely applied to single-phase induction motors.

An alternating magnetic field is defined by its change of intensity, the locality always remaining the same. This distinguishes it from a regular revolving field, which remains constant in intensity but revolves at a constant velocity. An alternating field can be replaced by two revolving fields rotating synchronously in opposite directions, both equal in strength, the latter being half that of the original alternating field. This resolution of the alternating into two revolving fields is correct only if the flux distribution of the former is sinusoidal, and, further, it is correct only if we wish to study inductive effects. The first of these conditions is not fulfilled in our case, but by introducing a correction we may put up with this inaccuracy. The inductive effect in our case is represented by the eddy current loss which, as pointed out, is larger than the hysteresis loss in practice, and we might, therefore, consider the whole of the core loss as following the same law as that of the eddy currents. Actually the hysteresis is proportional to the frequency and to $(\text{density})^{1\cdot6}$, the eddy currents to the $(\text{frequency})^2$ and $(\text{density})^2$, but as a whole the results obtained by considering the total loss as eddy-current loss only will give us an idea of what is actually happening. We will not go into the details of the mathematical proof, that resolving the alternating field into two revolving components is correct, but we can see at once that, when the two revolving fluxes of half the intensity of the original flux are in the same direction, they give the full value (Fig. 73), and that half a period later, when the alternating flux is zero, the revolving ones come into opposition, cancelling one another (Fig. 75).

When the armature is stationary (speed zero) only the teeth under the pole and the back of the armature take part in the hysteresis loss. We will take as a basis the core losses which would be caused in the armature if magnetised under these conditions and put

$$P_{c0} = C_0 \times v^2 \times B^2.$$



73 74 75

FIGS. 73–75.

B here stands for the highest density in teeth and back. The two component revolving fluxes have equal speeds under these conditions and cause the losses

$$P'_c = P''_c = C \times v^2 \times \left(\frac{B}{2}\right)^2,$$

or

$$P_{c0} = C_0 \times v^2 \times B^2 = 2 \times C \times v^2 \times \frac{B^2}{4}.$$

Therefore $C = 2 \times C_0$, and we find for the stationary armature :

Flux 1. Flux 2.

$$P_0 = 0{\cdot}50 \times C_0 \times v^2 \times B^2 + 0{\cdot}50 \times C_0 \times v^2 \times B^2.$$

Assuming that the armature revolves in the same direction as flux 1, the relative velocity between the latter and the armature diminishes as the speed increases, and becomes zero at synchronous speed; and, therefore, at this speed

$$P_{cs1} = 0{\cdot}50 \times C_0 \times (0)^2 \times B^2 = 0.$$

On the other hand, at synchronous speed the velocity relative to the armature of the flux 2, rotating backward, is double that at the beginning. Therefore

$$P_{cs2} = 0{\cdot}50 \times C_0 \times (2v)^2 \times B^2\,;\ P_{cs\,\mathrm{total}} = 2 \times C_0 \times v^2 \times B^2 = 2 \times P_0.$$

It follows that at synchronous speed the core loss is double that of the stationary motor, provided the flux is the same in both cases. If the speed is a times the synchronous speed the core loss is

$$P_c = P_0 \times (a^2 + 1). \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

For one-half synchronous speed we find $P_{c\frac{s}{2}} = 1{\cdot}25 \times P_0$.
,, five times ,, ,, ,, $P_{c5s} = 26 \times P_0$.

With increasing speed the core loss due to the alternation of the flux is of very small influence, a^2 becoming large compared with 1 in equation (17), both fluxes revolving then in the same direction relative to the armature at speeds not so very much different from one another. If the flux were constant (direct-current excitation) we could put

$$P_{cd} = C \times v^2 \times B^2 \times a^2 = 2 \times P_0 \times a^2,$$

provided that the maximum density B were the same in both cases. The ratio of the losses in the same armature with direct-current and alternating excitation, with the same maximum density, is therefore $g = \frac{2 \times a^2}{1 + a^2}$.

The following table shows g as a function of the speed:—

a.	g.	a.	g.
0	0	2	1·6
0·5	0·4	3	1·8
1	1	∞	2
1·5	1·38		

Above synchronism the core loss due to alternating excitation is smaller than that due to continuous excitation at equal flux densities. At very high speeds the ratio of the direct current to the alternating-current core loss approaches the figure 2. We can understand this relation, as with very high speed the alternation of the flux proceeds comparatively very slowly, and we can consider, for short intervals, the densities in which the armature revolves to be constant, the losses being at every moment $C_0 \times (v \cdot a)^2 \times (B_{\text{momentary}})^2$, *and therefore the mean value of the losses* $\frac{1}{2} \times C_0 \times v^2 \times a^2 \times B^2$. Now we know that with direct current we obtain a voltage $\sqrt{2}$ times as high as with alternating current, the maximum flux being the same in both cases. If we compare the losses caused by direct current and alternating fluxes, which are producing a given effective voltage in a certain armature, we find $g' = \left(\frac{1}{\sqrt{2}}\right)^2 \cdot g = \frac{a^2}{1+a^2}$, and consequently—

When giving the same voltage, the core loss in an armature is smaller when excited with direct current than with alternating current.

We could split up—though not quite correctly—the losses caused by the alternating-current flux into two portions: one caused by the alternation of the flux, which is independent of the speed, P_0, and the other part caused by the revolution of the armature, $P_0 \times a^2$. The former is made up for electrically by the magnetisation through the field coils, and causes a phase displacement between current and voltage in the latter; the part $P_0 \times a^2$ is overcome mechanically, and is felt as a reduction of the torque just as the friction in the bearings, or as the core loss in a direct current machine, P_{cd}. We may therefore compare this loss P_{cd} with $P_0 \times a^2$, and we find that, *for equal effective voltages, the losses are equal.* For the same maximum density, the core losses caused by revolving in an alternating flux are only half of those with direct current excitation.

This result gives the simplest way of calculating the core loss in an alternating current commutator motor where the maximum density has been found.

Calculate the "alternating" losses in field and armature as with a transformer (allowing for extra eddy-current losses) and calculate the "revolving" core losses as with a direct-current machine, and halve this amount.

The core losses caused by the armature reaction may approximately be estimated by calculating them just as if the main flux was not there, adding about 50 per cent. for the more unfavourable distribution of lines through the influence of the main flux.

We have developed general formulæ for the flux dependent on the output of the machine, utilising these formulæ in laying out a line of machines, providing sufficient section for the flux to pass, fixing approximate general dimensions in accordance with the output, and estimating from these the mean length of magnetic path. This can be done very closely if one keeps in touch with fully-designed machines—the core loss can be obtained quite generally as the function of the output, periodicity and number of poles. I have carried out this investigation for machines from 3 B.H.P. to 300 B.H.P. and found that, given the number of poles and frequency, *the core loss, taken as the percentage of the output, is almost independent of the output of the machine.*

Speaking in a very broad way, the condition that, to obtain sparkless commutation, the flux must not fall below a certain figure can be expressed in this form : *The core loss of the machine must not be less than a certain percentage of the output if the machine is to run sparklessly.* Naturally by saturating teeth, poles, back of armature and yoke very little indeed—*i.e.*, by making the dimensions very much larger than necessary, or by using specially good iron and by not filing the slots—smaller figures for the percentage core loss can be obtained than those I am giving in Fig. 76.

By employing commutating poles, the core loss may also be kept smaller, the losses in the copper rising at the same time.

For obtaining practical average figures, however, our data are near enough. Fig. 76 represents the "alternating" core loss only as a percentage of the output of the machine for different numbers of poles and its dependence on the frequency. We realise at the first glance to what enormous figures the core loss rises if the frequency exceeds 25 to 30 per second. This fact is easily understood, for to obtain sparkless commutation a certain flux must be employed regardless of the frequency, and this flux, fluctuating through the whole body of the machine, armature and field, must cause very high losses if the frequency of its fluctuation exceeds normal figures.

The high core loss is one of the factors—and not the most unimportant one—limiting the frequency permissible with alternating current commutator motors. It is very interesting to see that *the limit is about the same as that imposed, for instance, by the power factor and the losses in the resistance lugs.*

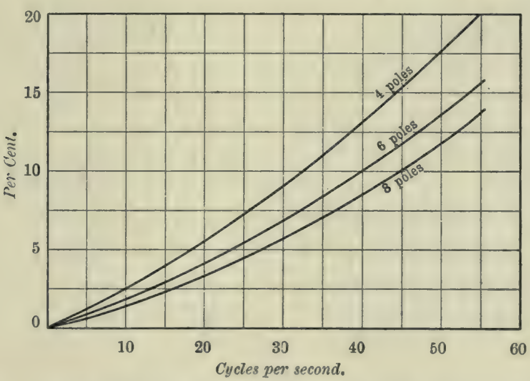


Fig. 76.—Core Loss with Stationary Armature (Alternating Core Loss) as Percentage of Output.

To calculate the total core loss we must first divide the "alternating" core loss, as given by our curves in Fig. 76, into two portions, that in the stationary and that in the revolving part of the motor. I have found that the ratio of core loss in the rotor to the total core loss varies very little indeed with a normal design. As an average it is about 20 to 30 per cent., say, 25 per cent. As the revolving core loss at synchronous speed is equal to the alternating core loss in the rotor, the total core loss when the machine is running synchronously (750 revs. per min. at 25 cycles, 4 poles) is 25 per cent. above those figures given in Fig. 76. With a machine for 25 cycles, 4 poles, running at 750 revs. per min., the alternating core loss is 7 per cent. of the output. At synchronous speed the revolving core loss is $0 \cdot 25 \times 7 = 1 \cdot 75$ per cent., the total $7 + 1 \cdot 75 = 8 \cdot 75$ per cent., say $\propto 9\frac{1}{2}$ per cent. of the total output, including losses through the armature reaction and eddy currents in the pole tips, &c. With 80 per cent. total efficiency the core loss would amount to $9 \cdot 5 \times 0 \cdot 80 = 7\frac{1}{2}$ per cent. of the input. With twice synchronous speed, 1,500 revs. per min., this figure would be $7 + (1 \cdot 75 \times 4) = 14$ per cent. of the output, or $16 \times 0 \cdot 8 = 12\frac{1}{2}$ per cent. of the input. The same machine, built for 750 revs. per min. 25 cycles, but having eight poles, would have an "alternating" core loss of 4·3 per cent. at 750 revs. per min.—*i.e.*, twice synchronous speed. In this case the revolving core loss would be $4 \cdot 3 \times 0 \cdot 25 \times 2^2 = 4 \cdot 3$ per cent., the total core loss; therefore, $4 \cdot 3 + 4 \cdot 3 = 8 \cdot 6$ per cent., and inclusive reactive core loss, say $\propto 9\frac{1}{2}$ per cent. The total core loss at 750 revs. per min. is in this case the same for 4 and 8 poles. At 50 periods, 4 poles, 750 revs. per min. (half synchronous speed), we find the "alternating" core loss 17·5 per cent., "revolving" core loss 1·1 per cent., total 16 per cent. of input. At 50 periods, 8 poles, 750 revs. per min. (synchronous speed), the "alternating" core loss is 11·5 per cent., the "revolving" core loss 2·9 per cent., the total $12\frac{1}{2}$ per cent. of input.

11. The Remaining Losses.

A. *The Friction Losses.*—The friction losses of an alternating current motor are naturally to be calculated in exactly the same way as those of any other machine. They consist of

1. Bearing friction.
2. Windage.
3. The brush friction on the commutator.

We should go altogether beyond the scope of this treatise if we were to enter into the theory of friction generally. It is, however, essential that we should have some basis for taking account of the friction losses when making out diagrams and estimates of the efficiency. For this purpose we will use the curve Fig. 77. This curve is only correct in a very general way; it gives average figures for friction of bearing and windage at different speeds. The commutator friction naturally cannot be made dependent on these two factors alone, as the number of brushes and the brush pressure have the greatest influence; and this number, again, depends on the current and the current-density. The commutator diameter and the class of the brush material has also considerable influence on it.

Assuming a current-density of 6½ amperes per square centimetre, a coefficient of friction of 0·30, and a pressure of 0·22 kg. per square centimetre, we obtain $\frac{1}{50}$ kg. friction per 1 ampere. With alternating current motors the commutator has a diameter of the order of 80 to 90 per cent. of the armature diameter. The latter, again, is pretty closely fixed by horse-power and speed, being only slightly dependent on the width of the armature if varied moderately. Resorting to practical experience on machine dimensions, I have developed a set of curves (Fig. 78), which will enable us to estimate approximately the brush friction of the motor without going into the design of the machine itself. The curves represent the brush friction loss per hundred amperes taken as a percentage of the output of the machine for different speeds and horse-power. They refer to machines for continuous rating; machines with intermittent rating have about 25 per cent. less losses than given by Fig. 78.

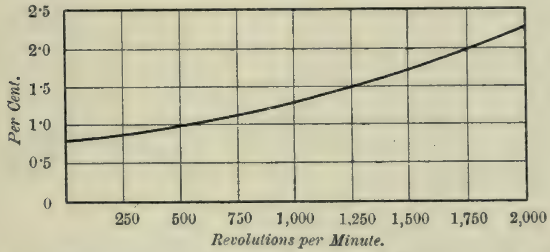


FIG. 77.—BEARING AND WINDAGE LOSS AS PERCENTAGE OF OUTPUT.

A general idea of the percentage brush friction loss can be gained from the curves (Fig. 79), which have been drawn up for 200 volts, the power factor and the efficiency having been assumed as 100 per cent. These figures require correction in inverse proportion to the actual voltage and actual efficiency and power factor. Thus a 250 volt motor for 200 H.P. at

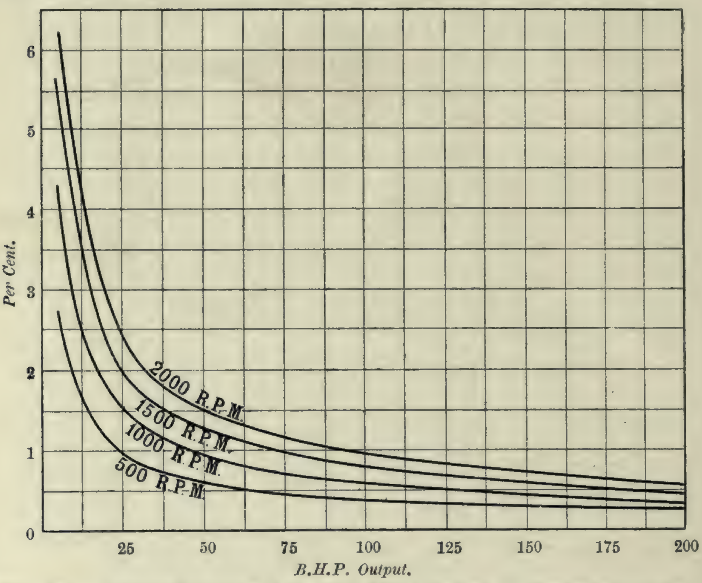


FIG. 78.—BRUSH FRICTION LOSS PER 100 AMPERES AS PERCENTAGE OF OUTPUT.

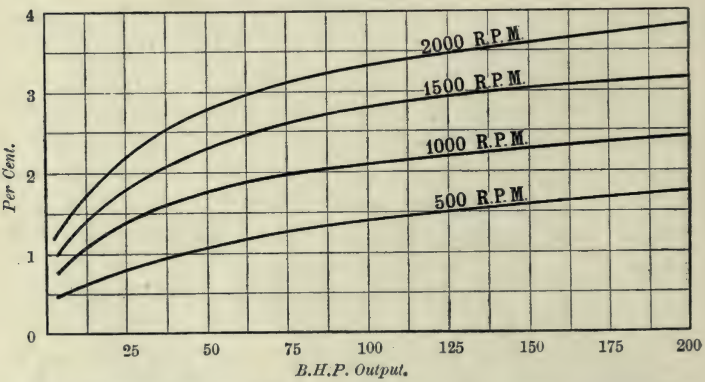


FIG. 79.—BRUSH FRICTION LOSS AT 200 VOLTS., APPARENT EFFICIENCY = 100 PER CENT.

750 revs. per min., with a power factor of, say, 90 per cent. and an efficiency of 85 per cent., would have a brush friction loss of about

$$2{\cdot}04\times\frac{200}{250}\times\frac{1}{0{\cdot}9}\times\frac{1}{0{\cdot}85}=1{\cdot}84\times\frac{1}{0{\cdot}85}=2{\cdot}15\text{ per cent.}$$

of the output, or 1·84 per cent. of the input.

B. *The Copper Losses.*—The copper losses consist of

1. Losses in the armature copper.
2. Losses in the compensating winding copper.
3. Losses in the field copper.

Copper losses in the armature can be obtained in a general way by a somewhat complicated process. The dimensions of the armature being approximately got by going into the design, the amount of copper to be accommodated on the surface can be found, leaving sufficient section for the teeth to carry the necessary flux. Further, the current this copper can carry is fixed by the cooling surface. Though the method is crude it is good enough to show the law which connects the copper losses with the output, the more as I have kept sufficiently in touch with completely designed machines. The curves, Fig. 80, are the result of my investigation, giving the copper loss in the armature as a percentage of the output of the machine for different capacities and speeds, the machine having four poles.

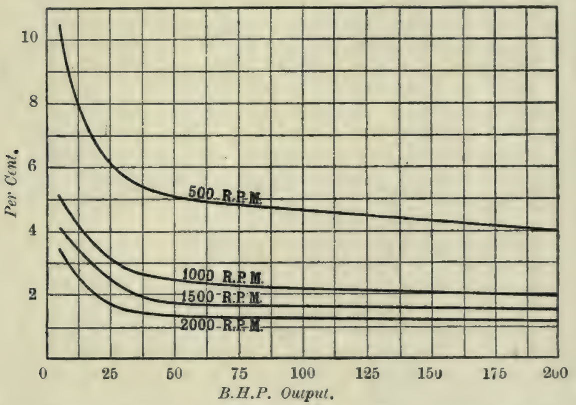


FIG. 80.—COPPER LOSS IN THE ARMATURE AS PERCENTAGE OF OUTPUT.

For a different number of poles we have to multiply by a constant, which is the smaller the higher the number of poles—*i.e.*, the shorter the end connections of the armature. This constant is: For six poles, 0·80; for eight poles, 0·67.

The losses in the compensating winding are closely related to those in the armature, both ampere-turns being almost equal. This winding being stationary, however, less losses are permissible. We may say, approximately, that

$$\frac{\text{Copper losses in compensating winding}}{\text{Copper losses in armature winding}} = 0{\cdot}75.$$

The losses in the main field coils depend very largely on the air-gap, but, as we saw from our former investigations on the power factor, the field ampere-turns must not be excessive compared with the armature ampere-turns, and, therefore, the loss in the field coils has some relation to the armature copper losses. With small machines the air-gap must—for mechanical considerations—be comparatively larger. We may say that with 5 H.P. to 10 H.P. machines the field loss is about 80 per cent. of the armature copper loss; with 100 H.P. to 200 H.P. machines it is only about 60 per cent.

12. The Efficiency.

In the literature on single-phase commutator motors elaborate studies can generally be found on the power factor, which figure is comparatively easy to determine, and which permits itself to be expressed by simple mathematical formulæ or by diagrams. Of equal—if not greater—importance is the efficiency, for which, in the previous chapters, preliminary studies of the losses were made. It would have made our work very bulky if the details of these developments had always been included, for it would have meant going into the law of heating and other questions which might be dealt with in a separate treatise as large as the present one. It is quite impossible to give in a general way efficiencies otherwise than approximately, and it is sufficient to form a general idea how these figures come out with the machines in question.

We will show the application of our tables and curves by an example.

What is the efficiency of a single-phase commutator motor for 50 h.p. 200 volts 25 cycles 750 revs. per min., having four poles and four brush arms, the power factor being approximately 0·90 ?

We find

Alternating core loss	7·00	per cent.
Revolving core loss	1·75	,,
Bearing friction	1·15	,,
Brush friction	1·95	,,
Commutator connections and brush drop	3·35	,,
Armature copper loss	3·50	,,
Compensating copper loss	2·50	,,
Field copper loss	2·10	,,
Total losses	23·30	,,
Output	100·00	,,
Input	123·30	,,
Efficiency	81·10	,,

The losses of a direct current machine with the same output and running at the same speed would be approximately as follows : —

Alternating core loss	0·00	per cent.
Revolving core loss	2·05	,,
Bearing friction	1·15	,,
Brush friction	1·75	,,
Commutator connection	0·15	,,
Brush drop	1·35	,,
Armature copper loss	3·30	,,
Compensating copper loss	0·00	,,
Field copper loss	4·85	,,
Total losses	14·60	,,
Output	100·00	,,
Input	114·60	,,
Efficiency	87·20	,,

As another example we may consider a very small machine, say a 5 H.P. motor for 25 cycles 1,500 revs. per min. 100 volts, with four poles and two brush arms, the power factor being assumed as 0·82. The losses are :—

Alternating core loss	7·0	per cent.
Revolving core loss	6·8	,,
Bearing friction	1·7	,,
Brush friction	2·6	,,
Commutator connections and brush drop	6·7	,,
Armature copper loss	4·0	,,
Compensating copper loss	3·0	,,
Field copper loss	3·5	,,
Total losses	35·3	,,
Efficiency	74·0	,,

The losses of a direct current machine would be :—

Alternating core loss	0·00	per cent.
Revolving core loss	8·00	,,
Bearing friction	1·70	,,
Brush friction	2·50	,,
Commutator connections	0·05	,,
Brush drop	0·45	,,
Armature copper loss	4·00	,,
Compensating copper loss	0·00	,,
Field copper loss	7·00	,,
Total losses	23·70	,,
Efficiency	81·00	,,

We see that in our examples with the alternating current motor the efficiency is about 6 or 7 per cent. lower and the losses are about 50 per cent. higher than with a corresponding direct current machine. This is chiefly due to the alternating core loss and the loss in the commutator connections.

13. The Diagram of the Real Motor.

The diagram of the real motor distinguishes itself from that of the ideal machine by taking account of the losses, but it has one weakness in common with the latter—the saturation of the magnetic path is supposed to be so low that the flux can be put practically proportional to the current. We must divide the losses into two classes :—

1. Losses which are proportional to the square of the current.
2. Losses which cannot be expressed as a simple function of the current.

The first class comprises all the copper losses in field, armature and compensating winding. We may also include the loss caused by the main current in the commutator connections and under the brushes, though the latter is not exactly proportional to the square of the current. One might feel inclined to let also the alternating core loss pass as belonging to the first class. This would, however, mean misunderstanding the character of the hysteresis and the eddy-current loss. They cannot be represented by substituting an equivalent resistance in the circuit of the main current; on the contrary, they apparently diminish the resistance of the main circuit of the machine, acting just as the secondary circuit of the transformer reacts on the primary. Therefore, the case is represented by an equivalent connection diagram (Fig. 60) with an ohmic resistance in parallel to the field. These losses turn part of the magnetising current into watt current; the resistance in the main circuit would produce a watt component in the voltage. Both make the angle between volts and amperes smaller, but the resistance of armature, field coils, &c., may seriously affect the overload capacity of the machine, reducing the voltage left for the armature. The alternating core loss has not this effect. Just like that of the eddy currents caused by the alternations of the flux is the effect of the circulating current in the coil under commutation, in fact it can be treated as part of and together with the core loss. The revolving core loss and the friction loss tend to prevent the armature from revolving; they diminish the useful torque. These losses are, therefore, covered mechanically and do not affect the phase of the current in any other way than the useful load on the pulley or pinion of the motor. As a fundamental rule for the design of the diagram, we will establish the following as the simplest way for practical purposes:—

We will construct the diagram, neglecting all losses of the second class, taking account of them afterwards by means of a correction. Then we can put in these losses more exactly than it would be possible if we should attempt to find an approximate law for the relation of loss and current so as to embody them in the diagram. In the linear diagram which we may draw up for a certain working condition, the effect of hysteresis, eddy and circulating current losses may easily be taken into account. We start with the assumption of a certain flux, OA (Fig. 81), requiring a magnetising current, OB, for driving it through the air-gap and other parts of the magnetic path in phase with the flux itself. The flux produces eddy currents, OC, and circulating currents lagging behind the flux by about 90 deg., which will be represented in the main circuit, together with the hysteresis losses, by the component BD vertical to OB (parallel to OC). OB and BD together give the main current OD, which flows through field and armature in series. The flux produces in the field coils an E.M.F. vertical to it, which is compensated by the E.M.F. $(-E_L) = OF$ vertical to the current and leading the latter by 90 deg. In phase with the current an E.M.F., OG, is required equal to current × resistance of field coils, so that the total voltage of the terminals of the latter is the vector sum of OG and OF—*i.e.*, OH.

The armature revolving in the flux OA generates an E.M.F., OK, in phase with OA. We add to this KL = E.M.F. of self-induction in armature and compensating winding plus watt component to cover the core losses caused by the armature and compensating cross-flux. KL is, due to this watt component, less than 90 deg. in advance of the current.

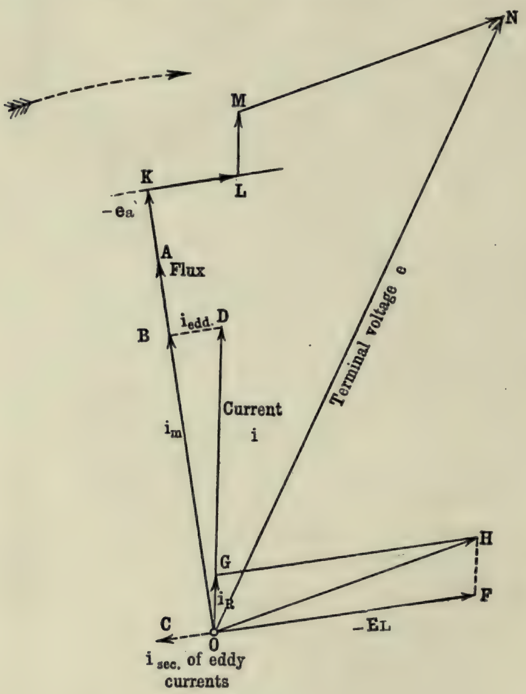


Fig. 81.

Adding further LM = current × resistance of armature, brush contact, resistance leads and compensating winding, we find

OM, the voltage across armature and compensating winding OM and OH together give the terminal voltage e=ON.

One of the interesting effects of the core, eddy and circulating currents loss is the shifting of the phase of E.M.F. E in the armature backward, together with the flux. E and the total current come out of phase, and the torque is slightly reduced, but there is also a beneficial effect, for through the shifting of phases the power factor is improved more than it would be in the ordinary way by the addition of the watt current due to hyteresis.

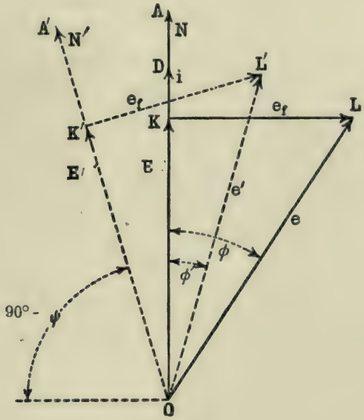


Fig. 82.

Special attention must be drawn to the fact that the harmful circulating currents reacting on the phase of the flux are quite useful, and for this reason they have sometimes been suppressed to a smaller degree than one might expect. Tests have been carried out by the Siemens-Schuckert Co. (Richter) to show that the power factor can be made equal to unity through allowing circulating currents to flow more or less unchecked. Naturally the heating of the brushes and the commutator prevent the actual practical application of this expedient.

The power-factor improving qualities of the losses caused by the alternations of the main flux and the shifting of E will be clearly seen from the simplified diagram Fig. 82. OKDAL is the diagram for the ideal motor. If now the flux N is made to lag behind the current i, by an angle, ψ (the hysteretic and eddy current lag), it will obtain the position OA′. The E.M.F. E will follow it to OK′ and, apart from details, the new triangle OK′L′ will be identical with the original one, OKL, the new phase displacement being $\phi' = \phi - \psi$. *The angle of the hysteretic and eddy-current lag subtracts directly from the original angle between current and terminal voltage.* Assuming, for instance, $\cos \phi = 0{\cdot}80$—*i.e.*, $\phi = 37$ deg.—and $\sin \psi = 0{\cdot}10$, $-\psi$ being 6 deg.—we find $\phi' = 37 - 6 = 31$ deg., and the new power factor $\cos \phi' = 0{\cdot}86$. Supposing we had a $\cos \phi = 0{\cdot}705$ only, ϕ being 45 deg. and $\sin \psi = 0{\cdot}10$, ψ is 6 deg. and $\cos \phi' = 0{\cdot}78$. This is only the improvement due to the shifting of the phase of the flux, a further increase of the power factor being effected by the watt-component of the current for covering the loss.

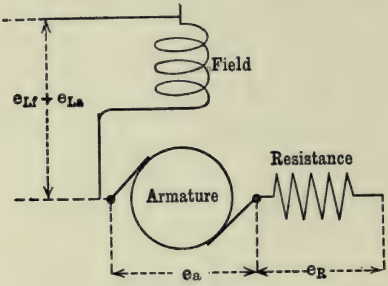


FIG. 83.

This diagram will be still clearer if we add together all components of voltages for overcoming the ohmic drop in field armature and brushes into one, in a way eliminating all resistance from the armature and field and adding it separately, as diagrammatically shown in Fig. 83. At the same time we combine e_{Lf} and e_{La}. Diagram Fig. 84 is the same as Fig. 81, except for the order in which the components of the voltage are combined. We find, again, the current-triangle $i_{\text{magn.}} - i_{\text{core loss}} - i$. In phase with $i_{\text{magn.}}$ we draw the flux and the voltage induced in the armature by rotation e^a. Vertical to it we draw $e_{Lf}+e_{La}$, and in phase with the total current i the ohmic drop $i\times r$. All three voltages together, e_a, $(e_{Lf}+e_{La})$ and $i\times r$, give the terminal voltage e.

If we do not take account of i_w in our diagram the effect in the combination of E.M.F.s is felt in $i\times r$ only, this factor being slightly reduced and shifted backward in its phase (Fig. 85). Except for this small defect we are able to construct a diagram completely neglecting i_w and to add this current component afterwards in a convenient way. The advantage of neglecting i_w at first lies in the fact that the E.M.F.s of self-induction of field, compensating winding and armature are 90 deg. displaced against i_m, not against the total current, and that E $(=e_a)$ is in phase with i_m only.

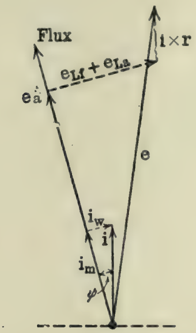


FIG. 84.

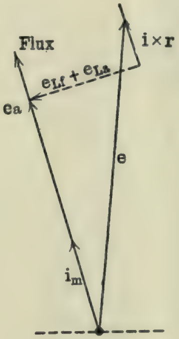


FIG. 85.

Except for the component $i\times r$ we could use our ideal diagram exactly, and it is clear that the presence of the resistance can involve only a minor modification.

Drawing again the circle and the triangle of voltages OAB (Fig. 86), AB now represents the E.M.F. of self-induction in field and armature $e_{Lf}+e_{La}$, OA is no longer $e_a=\mathrm{E}$ alone, but e_a plus $i\times r$. The phase of the current remains exactly the same—*i.e.*, that of e_a and $i\times r$, and, as before, its intensity can be represented by AB. Erecting again AC vertical to OB, BC is the torque and AC the watt component of the current, but no longer the useful energy, as it includes a component for the ohmic loss. When the motor is stationary e_a becomes zero and the point A obtains a position A′ on the circle not identical with O, OA′ being the drop in the ohmic resistance which cannot be eliminated. BA′ is the maximum current obtainable, the "short-circuit current," fixed by the self-induction of field and armature, and now, also, by the ohmic resistance. The short-circuit triangle OA′B, drawn for full voltage on the stationary motor, is a fixed quantity for a certain machine. Draw A′C′ vertical to OB; BC′ is the torque at starting and A′C′ the watt component of the current A′B, due to the ohmic losses in the motor. If the current is only AB, this watt component goes down to HC, as it can easily be proved that the amount HC, cut off from the vertical AC by the short-circuit line A′B, is proportional to the current AB. Therefore the part *AH between A′B and the circle represents the useful energy* except for the losses of friction and revolving hysteresis.*

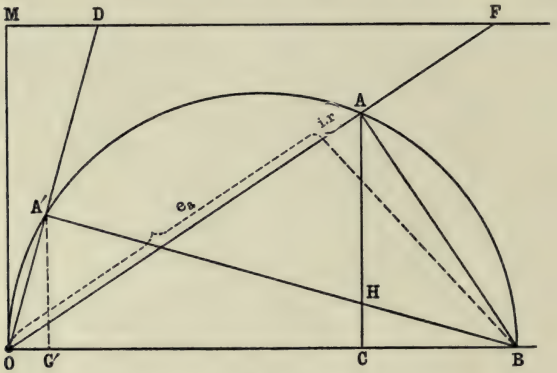


FIG. 86.

Drawing DF in Fig. 86 parallel to OB, similar to the diagram for the ideal motor, the amount cut off on this line by the lengthened OA—*i.e.*, DF—represents the speed. It is smaller than before by the amount DM, in consequence of the voltage drop in the ohmic resistance.

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* Compare *Electrotechnische Zeitschrift*, 1900, the Author's simplified form of the Heyland diagram for induction motors.

It remains to bring in the core loss component so as to fix the phase and amount of the total current. This can be done for every point of the diagram in the way shown in Fig. 87, i_w being determined as exactly as possible. Considering that the majority of the losses which i_w represents are eddy currents (including the circulating currents in this class), the component for these losses i_w can be put very nearly proportional to e_L,—*i.e.*, proportionate to i_m itself—the angle of hysteretic phase displacement ψ between i_m and i being assumed to be constant. Then i is equal to constant $\times$ i_m, so that it is only a question of "scale" for the current in our diagram to give the value of i direct. Also the influence the component i_w exercises on the power factor can be easily shown. AB being i_m (Fig. 88), we add i_w=AG at right angles (OA

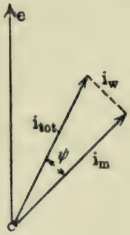


FIG. 87.

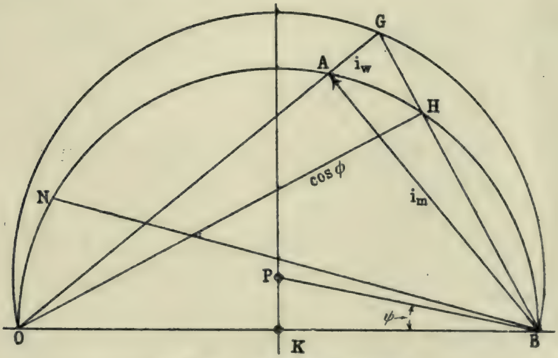


FIG. 88.—COMPLETE DIAGRAM.

lengthened); BG is then the total current i in proper phase, and OH is the power factor if OB=1. The geometrical locus for G is a circle which passes through O and B and the centre P of which is vertically above the centre of the main circle, the angle $PBO = \psi$.

14. Diagram, Tests and Performance Curves.

To draw up the diagram and determine the performance of an actual machine the following tests are required:—

1. *The resistance of field, compensating winding, armature, including resistance lugs and brushes.* (N.B.—The test can be taken with direct current.) It is advisable to drive the armature round very slowly (field not excited) so as to obtain a good average for the resistance of the lugs. The resistance ought to be measured with currents of various strengths, the direction ought to be changed as well as the direction of rotation, and the average value of the resistance measured should be used.

2. In connection with above test (1): *Tests of inductive drop and measurement of losses in armature plus compensating winding.* The armature and compensating winding should be excited in series with alternating current, the field being left unexcited and the armature revolving very slowly. Find the approximate laws: A. Volts on armature plus compensating winding=constant × amperes. B. Core loss caused by the stray flux = constant × $(\text{current})^2$.

3. *The no-load test.* Excite the field winding only, drive the armature at about normal speed, measure amperes and watts consumed by the field at different voltages. Measure voltage on armature at the same time. This test ought to be repeated with the brushes lifted, if it is desired to separate the energy consumed by circulating currents from hysteresis and eddy current loss. Repeat the test at a very slow speed indeed, and take average figures for the losses and currents. Try to obtain the laws:

(*a*) Inductive volts on field (including inductive drop and armature + compensating winding) equal to constant × amperes (as a crude average only) (Fig. 89).

(*b*) Volts on armature = constant × amperes × revs., or = constant × volts on field × revs.

(*c*) Core loss (including that due to armature and compensating winding, only a small amount) + circulating currents – ohmic losses in the field = function of current. Obtain an average figure for the hysteretic phase displacement ψ,

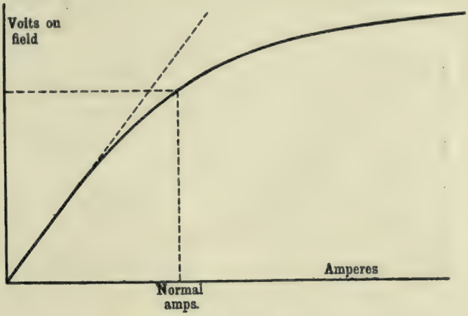


FIG. 89.

$$\sin\psi = \frac{\text{watts}}{\text{current} \times \text{inductive volts on field, arm. and comp. wdg.}}$$

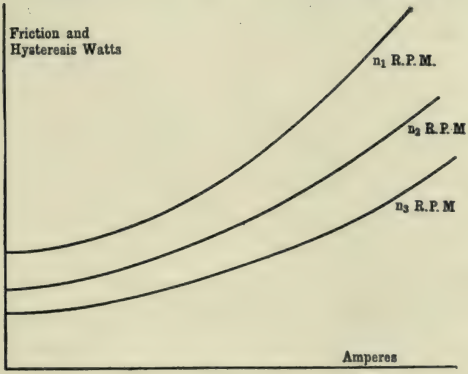


FIG. 90.

When taking above test measure friction loss and revolving core loss at different speeds and different currents and speeds respectively. Curves similar to Fig. 90 may be plotted, unless it is preferred to draw friction loss and core loss separately. To simplify the core loss and friction test, somewhat at the cost of exactness, one may take a single test instead of the set of curves. This test would be carried out with the motor running light, armature and compensated winding being shunted by a variable resistance, the total volts being kept constant (Fig. 91). The inexactness is caused by the drop in the armature, being due to a very small current only, and therefore the losses are measured slightly higher than their actual value.

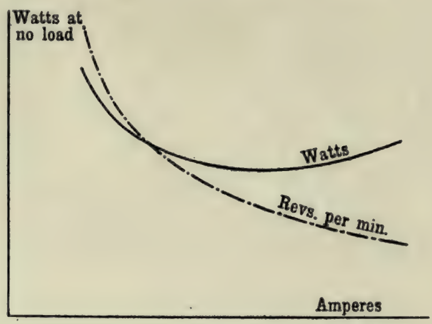


FIG. 91.

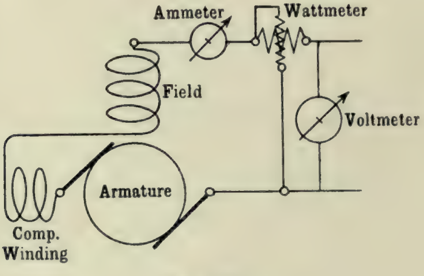


FIG. 92.

No results obtained by means of a diagram are as exact as those actually measured, and the most important point of all—the commutation qualities of the motor—cannot be determined by anything else but the full load run (Fig. 92). If it is a question of guarantees, *I should not hesitate either to construct a diagram similar to Fig. 82 for every point of the working range, taking account of even the slightest saturation and the core losses exactly as measured.* There is still a wide field for the diagram left—*i.e.*, to study the performance of single-phase motors in a general way, and to obtain, quickly, results as to its overload capacity, starting torque, power factor, &c.

We will construct the diagram for a definite example :—

50 H.P. motor, 25 periods, 200 volts at about 800 revs. per min.

1. Resistances—	Field	0·012 ohm.
	Comp. winding	0·014 ,,
	Armature	0·018 ,,
	Brushes	0·010 ,,
	Total ohmic resistance	0·054 ,,

2. Alternating-core loss, including losses through circulating currents under the brushes and volts on field (ohmic drop in field deducted) :—

Amperes.	Watts.	Inductive field volts.	Sin ψ.
100	900	44	0·205
150	1,600	63	0·170
200	2,500	82	0·152
255	3,400	97	0·137
350	5,200	113	0·132
500	8,600	128	0·134

3. Friction loss and revolving core loss at 200 volts constant (*see* Fig. 91) :—

Amperes.	Watts.	Revs.
150	3,100	1,280
200	2,350	950
255	1,800	770
300	1,600	680
400	1,400	590
500	1,450	520

Striking a broad average for currents between 150 and 350 amperes, we may put roughly:

$$\frac{\text{Inductive volts}}{\text{Current}} = 0{\cdot}37\,;$$

$$\text{Sin }\psi = 0{\cdot}148.$$

$$\text{Total impedance} = \sqrt{0{\cdot}37^2 + 0{\cdot}054^2} = 0{\cdot}372.$$

$$\text{Short-circuit current } \frac{200}{0{\cdot}372} = 540 \text{ amperes.}$$

$$\text{Sin }\psi \text{ at ideal short-circuit } \frac{0{\cdot}054}{0{\cdot}372} = 0{\cdot}146.$$

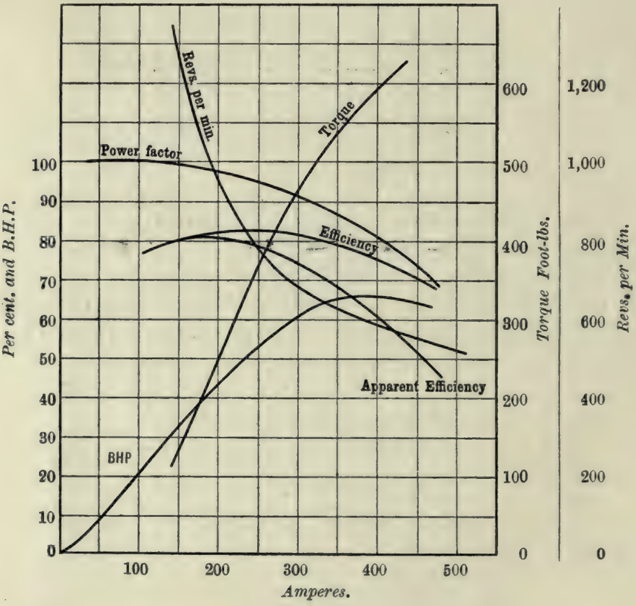


FIG. 93.

Construct the diagram, proceeding as follows (Fig. 88):—Draw OBN so that cos OBN=0·146; make BN=540 A, to suitable scale. Draw from centre on OB a circle through B and N. PK vertical on OB; sin BPK=0·148. From P as centre draw second circle through B, then any current line, say BG=255 amperes, has its proper phase, cos ϕ being given by OA (BO = 1). Torque, power, &c., being measured as described in the previous chapter.

The curves (Fig. 93) show torque, speed, power factor, horse-power output and efficiency as functions of the current, as obtained from the diagram. If there were no losses and no hysteretic phase displacement, the power factor at normal load (about 255 amperes) would be about 0·90, instead of 0·95. The improvement of the power factor "through the losses" is quite remarkable, and enables us to obtain power factors of 0·8 to 0·85 with quite small machines of 5 H.P. to 10 H.P., even for 50 cycles. It is worth noting that, with about 100 amperes, the power factor reaches unity, and, with still smaller currents, the machine supplies leading current to the line. This is due to the hysteretic phase displacement.

CHAPTER III.

THE THOMSON REPULSION MOTOR.

15.—The Induction of the Current in the Armature.

We have been treating in detail the simple series commutator motor—as a matter of fact, a machine in principle the same as the direct current machine, only supplied with alternating current. We have seen that the alternating current brought some disadvantages with it, such as reduction of output, power factor and efficiency. We have not utilised, however, the qualities prominent with alternating current, namely, the ability to transmit energy from one circuit to another without any electrical connection. This quality has been made use of with commutator motors chiefly in transforming the current for the armature. We may, for instance, wind the field for high voltage and small current and, by means of a transformer, transform the voltage down to a lower figure for the armature. This is indicated in Fig. 94. It is evident that the behaviour of such a motor would be exactly the same as that of an ordinary series machine, primary and secondary

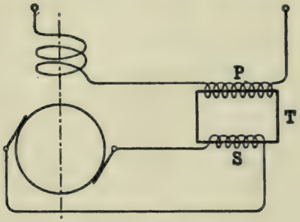


Fig 94.

current being kept rigidly in phase by the interlinking flux. Assuming for a moment that the transformer had a ratio of transformation 1 : 1, its introduction would not have the slightest effect on the behaviour of the machine. Hardly any advantage could be claimed for this arrangement, except that the heavy armature current need not be handled in the control apparatus, or that the voltage and current of the armature may be chosen as is most convenient, the field perhaps being wound for the high voltage of the supply. Compared with the transformation of the whole energy supplied to the motor, as shown in Fig. 95, the former method would have the advantage of requiring a somewhat smaller transformer, but, on the other hand, the field winding must be a high voltage one.

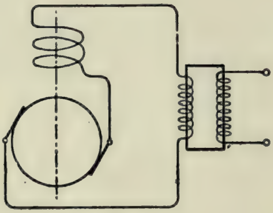


FIG. 95.

Looking at the armature circuit of Fig. 94 we find that it is short-circuited in itself electrically, and not connected with the primary at all. Now, it is possible to carry out the transformation in the motor itself instead of in the transformer, utilising part of the main flux, which produces the torque, for the transformation; we could, as it were, combine motor and transformer. In this case the brushes would be simply short-circuited, the secondary winding S of the transformer dropping out, and we should arrive at a commutator motor of the induction type, the "repulsion motor."

Starting from an ordinary series motor, we see that, apart from the circulating currents produced in the coils under commutation and short-circuited by the brushes touching several segments at the same time, the field has no inducing effect on the armature winding through the alteration of its intensity. Even if we short-circuit the armature, as proposed (Fig. 96), exciting the field up to its full value, the stationary armature would not have any current induced in it. The conductors from *a* to *b* (Fig. 96) receive E.M.F.s in opposite directions to those induced in the conductors from *b* to *c*, and both cancel one another. Matters would be different if the brushes were shifted 90 deg. (Fig. 97). In this case the E.M.F.s from *a* to *b* and *b* to *c* would add up, and the alternating flux would induce a very heavy current in the short-circuited armature. But under these conditions no torque whatever would be developed, as the dynamic actions between the flux issuing from the poles and the conductors from *a* to *b* and *a* to *d* would cancel one another.

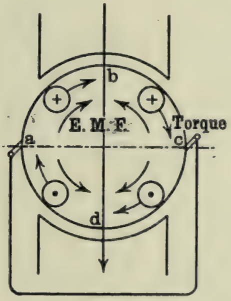


FIG. 96.

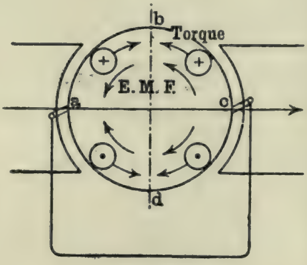


FIG. 97.

With the field shifted 90 deg. from the brush line (Fig. 96), brushes in the neutral zone, the field can only exercise torque on the armature winding, but no induction of current through the alternation of the flux—*i.e.*, no transformer action—can take place. In the case of coincidence of the field line and brush line it is just the reverse. The arrangement we should have to adopt would be a combination of the two, as shown in Fig. 98. Starting from the ordinary motor, with the brushes in the neutral position and short-circuited, we provide a second pair of poles, 90 deg. displaced from the first pair and excited in series with the latter. Pair number 2 would induce the current in the armature and this current would create a torque with the flux produced by pair number 1. Compared with Fig. 94 the field winding 2 would be the primary P of the transformer T, and flux N_2 the transformation flux, and the armature would be equivalent to the secondary S. Fig. 98 actually represents the diagram of the Atkinson repulsion motor.

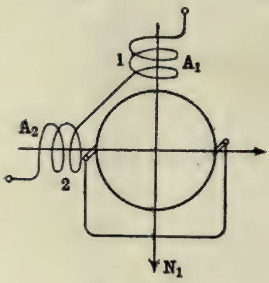


Fig. 98.

16.—The Repulsion Motor of Elihu Thomson.

The practical form of such a machine would naturally be different from that shown diagrammatically in Fig. 98, first of all because it would be difficult to provide room for the two pairs of poles and the two windings side by side. The neutral zone will, through the presence of the poles marked 2, more or less disappear, as the space must be used for the flux N_2, inducing the current in the armature. The form of the stator would

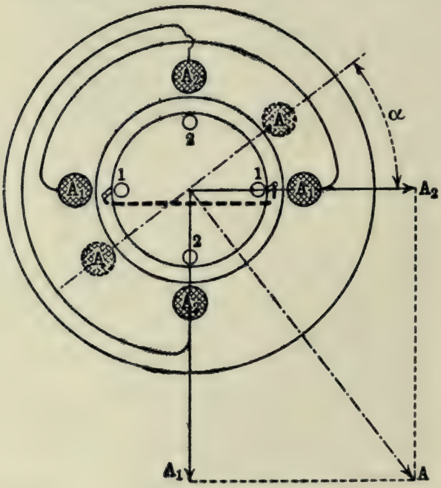


Fig. 99.

therefore be very similar to that of an induction motor with a two-phase winding, one locally displaced from the other by 90 deg., as shown in Fig. 99. Going still a step further, we recognise that at any moment under consideration in a certain stator it is only in imagination that there can be two magnetising forces. In reality we can join the two windings into one. The resultant winding A will have a position somewhere between windings A_1 and A_2 according to the strength of either. Considering, therefore, this one winding only as existing, we find that the arrangement is the same as that shown in Fig. 96, with the exception that the interpolar space has disappeared and that the brushes are shifted from the neutral position. We have now developed the repulsion motor in its original form, as invented by Elihu Thomson.

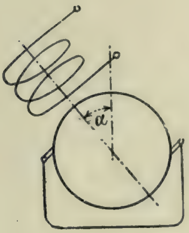


Fig. 100.

For theoretical consideration, and to understand its action, it is by far the most convenient way to remember that the single winding on the stator can be split up into two components, the centre line of the first A_1 being shifted 90 deg. from the brush line and producing the flux for exerting the torque. The centre line of the second component A_2 coincides with the brush line, and A_2 serves to induce a current in the short-circuited armature. If the field winding is displaced by α deg. from the brush line (Figs. 99 and 100), and A is the number of ampere-turns of the resultant field winding we can put

$$A_1 = A \times \cos\alpha \quad \text{and} \quad A_2 = A \times \sin\alpha.$$

$$\frac{A_1}{A_2} = \tan\alpha\,; \quad \text{for } \alpha = 45\,\text{deg.}, \ \frac{A}{A_2} = 1.$$

These are approximate only, and have by hypothesis a "sinusoidal" distribution of the field winding, a condition which is nearly fulfilled if the winding is distributed over several slots, similar to the compensating winding. This arrangement is not the most favourable with ordinary series motors, but it becomes more so with an induction type of motor, where the necessity of producing the armature current by means of transformation is important. For this reason the leakage between the inducing (primary) stator winding and the induced (secondary) armature ought to be kept as small as possible by making one a true picture of the other—*i.e.*, distributing the stator winding over many slots on the inner surface of the field. Fig. 99 shows clearly that a concentrated primary winding, A_2, would only induce effectively on conductors close to it (2), and only with great loss of lines on those closer to the brushes (1). From this point of view it would be best to provide two primary windings, one distributed for the induction and one concentrated for producing the main flux. In fact, the compensating winding of an ordinary series motor might be used as an induction winding. A method has been described (page 36) in which the compensating winding did not carry the main current at all, but was short-circuited in itself and a current produced by mutual induction from the armature. In our case, compensating and armature windings would simply exchange rôles, the latter being short-circuited and the former carrying the main current. There is, however, one essential difference between what happens when simply compensating the armature reaction and when transmitting the energy to the armature by mutual induction in the motor. In the former case only a very small flux interlinked both windings one with another; as a matter of fact, it is the object of compensation to keep the flux down. In our case the whole flux in the transformer T in Fig. 94 transmitting the energy to the armature must alternate in the direction 2.

17.—E.M.F.s, Currents and Fluxes in the Thomson Motor.

The electrical energy P_1 equivalent to the mechanical power is: Current × E.M.F., E_1, in the armature conductors produced by these conductors cutting the flux N_1. This will be so in whatever way the energy has been transmitted to the armature—if by conduction (series motor) or induction (repulsion motor). The energy transmitted to the armature conveyed by the flux N_2 is $P_2 = \text{Current} \times \text{E.M.F.}, E_2$, induced in the armature by flux N_2. P_1 is (apart from losses) equal to P_2, and therefore the E.M.F. induced in the armature by the "alternation" of the flux N_2 is equal to the counter E.M.F. due to the armature "moving in" N_1.

$$E_1 = E_2.$$

Actually $\quad E_2 = E_1 + \text{ohmic and inductive drop.}$

For both E.M.F.s the number of conductors is the same—that of the armature—the number of cycles at which N_2 alternates is fixed by the supply frequency, whilst the relative speed between armature and flux N_1 is fixed by the number of revolutions.

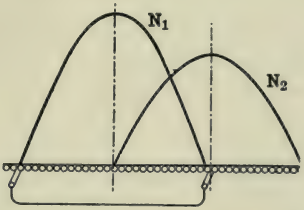


FIG. 101.

Fig. 101 represents the armature surface with its conductors. and N_1 and N_2 distributed sinusoidally. As has already been shown (formula 2, page 8),

$$E_1 = \frac{1}{\sqrt{2}} \cdot s_a \cdot \frac{n}{60} \cdot N_1 \cdot 10^{-8} \text{ volts.}$$

The calculation of E_2 requires a simple integration, as we have to deal with a winding spread over a circumference.

We find $\quad E_2 = \frac{1}{\sqrt{2}} \cdot s_a \cdot v \cdot N_2 \cdot 10^{-8}$ volts.

Consequently, as $E_1 = E_2$, $\frac{n}{60} \cdot N_1 = v \cdot N_2$,

or $$\frac{N_2}{N_1} = \frac{n/60}{v} \quad \text{and} \quad \frac{N_2}{N_1} = \frac{n}{n_s} = \frac{\text{actual speed}}{\text{synchronous speed}}.$$

At synchronous speed both fields would be equal. The higher the speed the larger the ratio N_2/N_1. If the speed were to alter inversely as N_1 (terminal voltage constant and small inductive drop in the field), N_2 would be constant.

Apart from the fact that N_1 and N_2 are contained in the same body, and the peculiarity that both act on the same winding, the characteristic of the repulsion motor ought to be the same as that of the series motor with transformer for the armature (Fig. 94)—*i.e.*, if the transformer is perfect (practically without leakage and magnetising current)—the same as in an ordinary series motor, because, "after the transformation," current and flux N_1 work together in the same way as we have stated in full detail with regard to the series motor. But unfortunately the transformation is *not* perfect, as it has to be carried out across an air-gap separating primary and secondary.

We might simplify the investigation, replacing our motor by two motors (Fig. 102), motor I carrying winding W_1, with the ampere-turns $A_1 = A \cos \alpha$ and flux N_1; motor II carrying W_2, with the ampere-turns $A_2 = A \sin \alpha$ and N_2. The former is the ordinary series motor, while the latter may be looked upon as stationary as it does nothing but transmit energy.

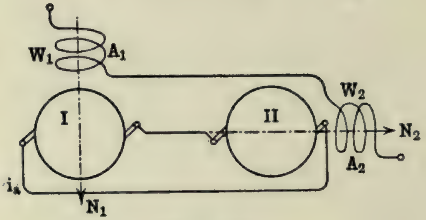


FIG. 102.

As already pointed out with regard to Fig. 94, the electrical equivalent of the transformation—apart from voltage and current variations which accompany it—is a direct supply of the energy to the secondary circuit. It is common knowledge that it is a matter of indifference to the supply circuit whether the energy is supplied to an apparatus through a transformer or direct, provided we take care of the magnetising current and the leakage if these figures are large, and furthermore provided that account be taken of the ratio of transformation. As first shown by Steinmetz, this can be effected by a combination of choking coils (Fig. 103), L_1 representing the self-induction of the primary, L_2 that of the secondary winding of motor II, L_3 being the choking coil taking the magnetising current to drive the flux N_2 through air-gap and iron. L_1 and L_2 act just as an increase of the field and armature leakage of an ordinary motor, so that we can include them in these figures. Its effect is an increase of E_L, and consequently a reduction of the power factor and overload capacity, but not any principal alteration in the performance of the machine. The rotor current i_a, flowing in z_a turns, must be multiplied by the ratio of transformation z_a/z_2 to obtain its primary equivalent, which is to be introduced into the diagram. *Therefore the repulsion motor is equivalent to an ordinary series motor with a choking coil in parallel with the armature* (Fig. 104), the magnetising current taken by the choking coil being as high as would be required to produce the transformation flux N_2 in the motor, the number of turns being the total multiplied by the sine of the brush angle α (Fig. 100).

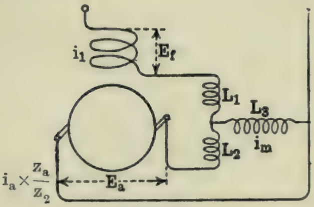


FIG. 103.

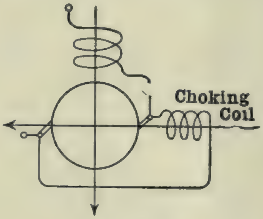


FIG. 104.

A consideration on the basis of our previous treatment will show us how large is i_m. At synchronous speed the two fluxes N_1 and N_2 are equal. The reluctances of their paths being the same, the *wattless energy to produce them will be the same.* Therefore

$$E_f \times i_1 = E_a \times i_m,$$

or $i_m = i_1 \cdot \frac{E_f}{E_a}$; for any other speed $i_m = i_1 \cdot \frac{E_f}{E_a} \cdot \left(\frac{n}{n_s}\right)^2$.

The diagram of the repulsion motor becomes very simple. In Fig. 105 we draw i_1. In phase with it the flux N_1 and the E.M.F., E_a, induced by the armature revolving in N_1. At right angles to i_1 we draw the E.M.F. of self-induction of winding W_1, E_f (inclusive drop in L_1 and L_2, Fig. 103), which is to be added to E_a in order to obtain the terminal voltage e. So far we have drawn the diagram of the primary circuit, which is exactly the same as that of the series motor. If there is only very little ohmic and inductive drop in the armature, E_a is its terminal voltage and that of the choking coil. So we find the current i_m in the latter 90 deg. behind E_a. The armature current i_a (actually multiplied by z_a/z_2) assumes such a value that i_a and i_m combine to the total current i_1. As $\frac{i_m}{i_1}=\frac{E_f}{E_a} \cdot \left(\frac{n}{n_s}\right)^2$ at synchronism, the angle between i_a and i_1 is the same as between e and E_a or i_1—that is, the angle ϕ itself, generally tan β tan ϕ. $(n/n_s)^2$. As $E_f=i_1 mL$ it represents i_1, the same as in the circle diagram for the series motor. For the sake of comparison, side by side with this diagram the diagram of the series motor has been drawn (Fig. 106). Starting again from the same value of $i=i_1$ we find $N=N_1$, just as large as before with the same E_a. *So the only difference lies in the alteration of the armature current, which is larger by the amount i_m added graphically to $i=i_1$.*

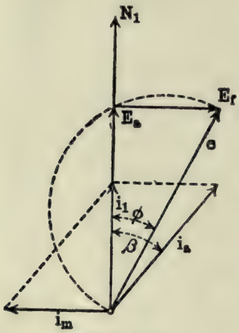


FIG. 105.

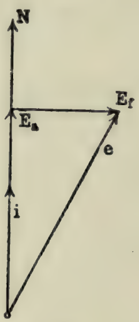


FIG. 106.

If we wish to study the performance of a repulsion motor we can use the same linear diagram and also the circle diagram as for the series motor.

Let us assume that we have a series motor, say without compensating winding, but with distributed primary coils. Referring to Fig. 105, we will assume $\cos\phi = 0{\cdot}85 = \frac{E_a}{e}$; $\frac{E_f}{E_a} = \tan\phi = 0{\cdot}73$, so that $\frac{\text{field ampere-turns}}{\text{working armature A.T. } (i_a)} \sim 0{\cdot}40$. This motor may, as an example, run under a load corresponding to the current I. Transforming this machine into a repulsion motor, we short-circuit the armature and shift the brushes from the neutral by a certain angle, α (Fig. 100). Ohmic drop may again be neglected. Fig. 98 represents the equivalent repulsion motor, A_1 having $S \times \cos\alpha$ turns and A_2 having $S \times \sin\alpha$ turns, if S is the number of turns of the series motor field winding.

Starting with the same current I as before, the ampere-turns for the production of the flux, as well as the latter itself, will now be smaller by $\cos\alpha$ (*i.e.*, $N_{1r} = N_s \times \cos\alpha$), and the E.M.F. E_{fr} of the field (*i.e.*, now that on the $S \times \cos\alpha$ turns, which is proportional to both flux and turns) will be equal to E_{fs} of the series motor $\times \cos^2\alpha$. The index s refers to series, index r to repulsion motor. Through shifting the brushes we have reduced the flux-producing portion of S, since part of it, namely $S \times \sin\alpha$, is split off for transmitting the energy to the armature. Now, since e, E_f and E_a (Fig. 105) form a right angled triangle, and E_{fr} decreases with increased brush shifting, E_a (*i.e.*, the voltage on the $S \times \sin\alpha$ turns) becomes larger and larger, and approaches the terminal voltage e, the latter being kept constant; E_a rises and N_1 falls with increasing α. Therefore the speed will go up the more the brushes are shifted from the neutral position. This affords a very convenient means to regulate the speed and even to reverse the motor by shifting the brushes forward instead of backward. Applying the curtain rule to Figs. 107 and 108 the law can be easily deduced that *the brushes of the repulsion motor must be shifted opposite to the desired direction of rotation.* One has to bear in mind that conductor C of the stator (primary of transformer) induces a current of opposite direction in the armature conductors closest to it (secondary of transformer).

The $I \times S \times \sin\alpha$ ampere-turns of the primary transformer winding will be reflected by the component i_a of the armature current. The ratio $\frac{\text{field-turns}}{\text{armature-turns}} = 0{\cdot}40 \times \sin\alpha$. Therefore

$$i_a = I \times 0{\cdot}40 \times \sin\alpha.$$

The useful rotor current i_a is 0 if the brushes are in the neutral zone ($\alpha = 0$), and it reaches its highest possible value for $\alpha = 90$ deg, namely $0{\cdot}40 \times I$.

It will be checked still more in a real machine containing self-induction and resistance in the armature. *So we find that the armature receives as a maximum only* 40 *per cent. of the current it has been designed for.* Since an angle $\alpha = 90$ deg. is naturally impossible, as no ampere-turns $I \times S \times \sin\alpha$ for the production of the flux N_1 would be left, the rotor current would be actually less than 0·40, or about one-third or one-fourth of the current it is capable of carrying. Without proceeding further with this investigation, we can conclude that *the field winding of the repulsion motor must be stronger than that of the series motor, compared with the armature winding, because a very strong component of the field ampere-turns is split off to induce the armature current.*

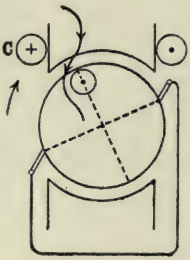


FIG. 107.

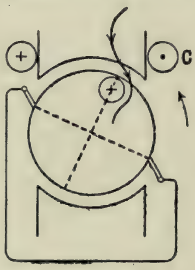


FIG. 108.

We can easily see what the ratio, field ampere-turns/armature ampere-turns, will be in our case. To produce the same flux $N_1 = N$ and to obtain the same speed and power factor as with the series motor

$$A_{f1} = A_f \times \cos\alpha = 0{\cdot}40 \times A_a,$$

A_f = stator-ampere-turns,

A_a = rotor-ampere-turns (working component)

Further $\quad A_{f2} = A_f \times \sin\alpha = A_x.$

A_a is the useful ampere-turns not including the magnetising component i_m. One might for a moment assume that the air-gap was so small that $i_m \rightarrow 0$, then

$$A_f = \sqrt{A_{f1}^2 + A_{f2}^2} = 1{\cdot}08 \times A_a.$$

Even if A_{f1} was as high as $0{\cdot}70 \times A_a$, which would give a very bad power factor, we should only have $A_f = 1{\cdot}22 \times A_a$, so we may say that *with a repulsion motor the field ampere-turns are about* 10 *to* 20 *per cent. higher than the working component of the armature ampere-turns.*

While the component A_{f1} *is equal to the ordinary field winding of the series motor* (which in this case we must assume to be a distributed one), A_{f2} *is just as large as the compensating winding,* namely, equal to the armature ampere-turns, so that the total ampere-turns on the field and the useful ampere-turns on the armature are exactly the same as with the series motor. With the series motor, too, it is quite possible to join field and compensating winding together.

We may, at the same time, fix the angle of brush shifting from the equation $\tan\alpha = \frac{A_{f2}}{A_{f2}}$; $\tan\alpha = \frac{1}{0{\cdot}4} = 2{\cdot}5$; $\alpha = 68$ deg.

The magnetising component of the armature current producing N_2 is, at synchronous speed, $i_m =$ watt component $\times \frac{E}{E_a} = 0{\cdot}73 \times$ watt component. The total armature current therefore becomes $\sqrt{1 + 0{\cdot}73^2} \times$ watt component $= 1{\cdot}24 \times$ watt component. The armature current is at synchronous speed 24 per cent., and the ohmic loss about 50 per cent. more than with the series motor, these relations being still worse for the repulsion motor at higher speed. The higher copper loss may naturally be met by providing a greater section of copper in order to reduce the losses for sake of heating—*i.e., by making the repulsion motor bigger.* With large motors for railways the increase of size cannot be afforded; with small motors one may sacrifice material for the advantage of speed regulation by brush shifting. Auxiliary apparatus for regulating the speed is superfluous with a repulsion motor. Even a switch in the primary circuit is unnecessary to make the motor tops, as we have only to shift the brushes 90 deg. against the polar line to effect this (Fig. 96). In this position there will be no circulating currents induced into the coils short-circuited by the brushes touching two segments, these coils not being exposed to the induction of the main flux.

18.—The Core Loss in the Thomson Motor.

It remains to determine the influence the transformation flux N_2 has on the core loss and the coils under commutation. N_1 and N_2 are displaced 90 deg. locally and in phase relation. If N_1 and N_2 are equal (synchronous speed) they would compound to form a proper revolving field of constant intensity, as in a two-phase induction motor (Figs. 109 and 110). If they are not equal, the revolving field is "elliptical"—*i.e.*, it alters its intensity when revolving (Figs. 109 and 111).

We may again attack our problem in the same way as we did with the series motor—*i.e.*, resolve each alternating field into two revolving ones of half the intensity. In our case we obtain two pairs of revolving fields :—

> From N_1, one with the intensity $B_{1f}/2$ revolving forward and a second one $B_{1b}/2$ revolving backward ;
>
> and from N_2 one with the intensity $B_{2f}/2$ revolving forward and a second one $B_{2b}/2$ revolving backward.

Now the two fields revolving forward add up, the two backward fields subtract each moment, as will be easily seen from Figs. 112 and 113. Fig. 112 represents the position of the fields at the moment when N_1 has reached its maximum value and N_2 is zero, and Fig. 113 the conditions half a period later when N_2 is at its height and N_1 zero. By studying further Figs. 114 and 115 it will become evident that the forward direction is the direction of rotation of the revolving armature, Fig. 114 representing the conditions of $N_1 = N_{1\text{max.}}$ and $I_1 = I_{1\text{max}}$, Fig. 115 half a period later, when $i_m = i_{m\text{max.}}$

We obtain a field of intensity, $B_1 + B_2/2$, revolving forward, the other one, $B_1 - B_2/2$, revolving backward. The speed of either is the synchronous speed n_s. The speeds relative to the

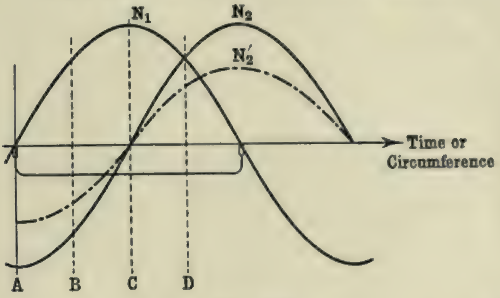


FIG. 109.

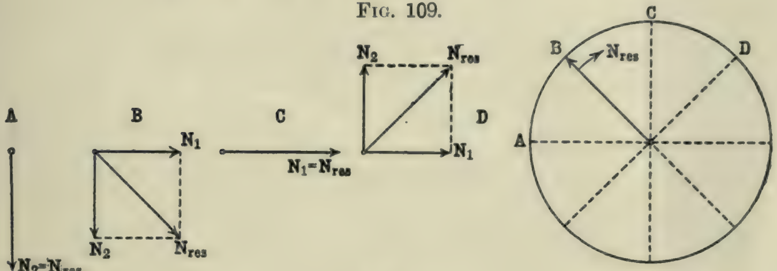


FIG. 110.

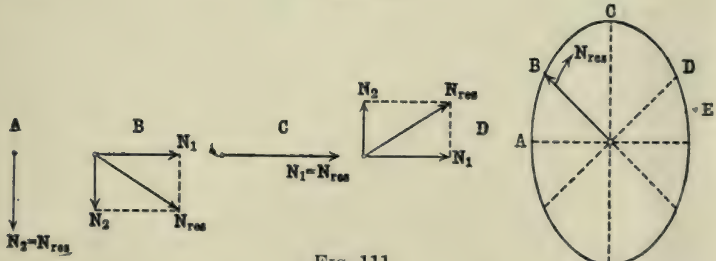


FIG. 111.

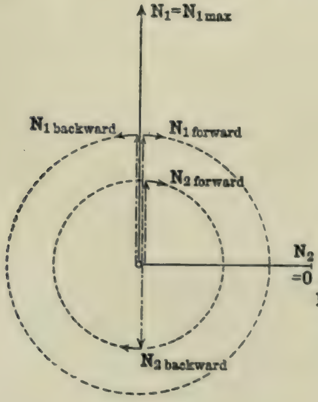


FIG. 112.

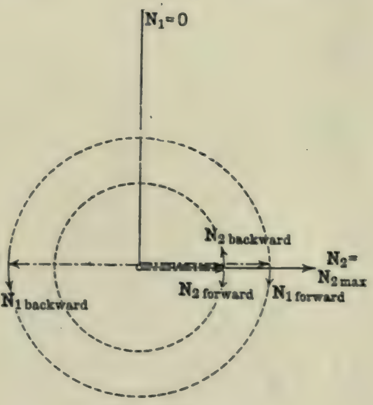


FIG. 113.

armature are $n_s - n$ and $n_s + n$ respectively; or, putting again $n = a \times n$, $n_s(a-1)$ and $n_s(a+1)$.

The core losses (chiefly eddy currents) caused by the two fields are

$$p_f = \text{constant} \times (B_1 + B_2)^2 \times (a-1)^2,$$
$$p_b = \text{constant} \times (B_1 - B_2)^2 \times (a+1)^2,$$

and $$p \text{ total} = p_f + p_b = \text{constant} \times 2(B_1^2 + B_2^2) \times (a^2+1) - 8a \,.\, B_1 \,.\, B_2.$$

The stator naturally has zero speed and thus $a=0$. Therefore the stator core loss is $\text{constant} \times 2(B_1^2 + B_2^2)$. For a series motor ($B_2 = 0$) only B_1 exists—*i.e., in a repulsion motor the stator core loss is higher than that in a series motor, the transformer flux* N_2 *and the motor flux* N_1 *each causing losses the same as if the other was not there.* Actually the losses will be somewhat smaller than this amount because we have assumed the whole core loss to be due to eddy currents.

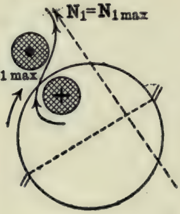


FIG. 114.

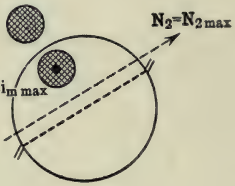


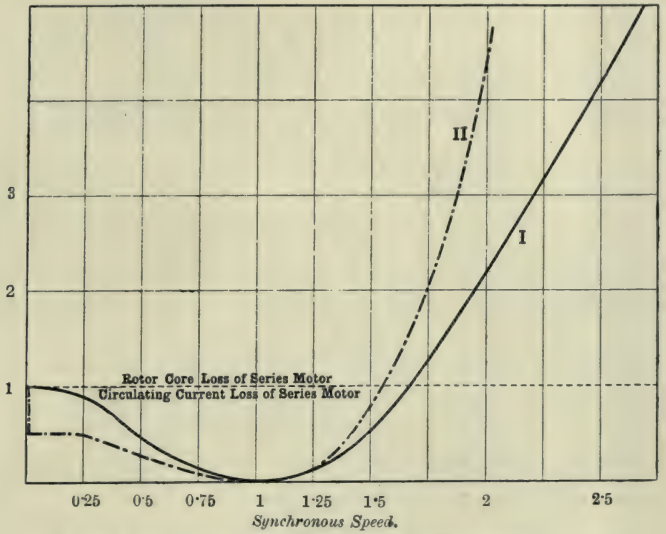
FIG. 115.

As to the *rotor loss*, we find for $a = \frac{\text{actual speed}}{\text{synchronous speed}} = 1$ and $B_1 = B_2$: $p_{\text{tot.}} = 0$. With the series motor it would be $\text{constant} \times 2 \times B_1^2$.

Generally we may put for a certain speed $n = a \times n_s$, neglecting saturation

$\frac{B_2}{B_1} = \frac{n}{n^s} = a$ and $p_{\text{tot.}} = \text{constant} \times 2B_1^2 \times (1+a^2)(a^2+1) - 8a^2 \times B_1^2$ $= \text{constant} \times 2 \times B_1^2 \times (a^2-1)^2$. With the series motor, $p_{\text{tot.}} = \text{constant} \times 2 \times B_1^2 \times (a^2+1)$. The ratio of the rotor core losses in a repulsion motor to those in a series motor $= \frac{(a^2-1)^2}{a^2+1}$.

Consequently in the repulsion motor the rotor core losses can be neglected for speeds in the proximity of synchronism. At very high speeds the rotor core loss of the repulsion motor is much higher than that of the series motor (*see* Curve I., Fig. 116).



I. Rotor Core Loss of Repulsion Motor at different speeds.
II. Circulating Current Loss of Repulsion Motor at different speeds.

FIG. 116.

19.—Circulating Currents.

The most interesting *effect of the transformation field* N_2 is that on *the circulating current in the coils short-circuited by the brushes.* N_2 is in the same position as was the interpolar space in the series motor, and therefore its lines are cut by the coil under short-circuit, and an E.M.F. induced in it. Moreover, the phase displacement between N_1 and N_2 is 90 deg.—that is, the condition necessary to counteract the circulating current induced by the alternation of N_1. It is most easy not to calculate for each field singly but to combine N_1 and N_2 to form two fields revolving backwards and forwards at synchronous speed in the same way as we did just now when studying the core loss question. Here we have to apologise again for carrying out the splitting up of the alternating field into two revolving ones with the series motor where the interpolar space makes the existence of proper revolving fields impossible.

Considering again the synchronous speed first, with the repulsion motor the field revolving backwards is 0 and the field revolving forwards has no speed difference relative to the rotor. Therefore, at synchronism, there is no circulating current induced in the coils short-circuited by the brushes. With the *series motor* the E.M.F. induced by the forward and backward component of the field is in each case constant × B_1. Therefore both together are constant × 2 × B_1, *irrespective of the speed*, the coil under commutation being always in the neutral zone, just as if it were part of the stator. The losses are constant × 4 × B_1^2. No cutting of lines through the movement takes place, as there is no field in the neutral. With the *repulsion motor* we may put the losses caused by forward and backward fields in the same way as with the eddy current losses, namely, $2 \times B_1^2 \times (a^2-1)^2$. The ratio of the losses in the resistance connections in the repulsion and series motor is

$$C=\frac{(a^2-1)^2}{2},$$

this relation being represented by Curve II., Fig. 116.

The circulating currents in the coils under commutation with the repulsion motor are, for speeds in the proximity of synchronism, either 0 or a small fraction of that of the corresponding series motor. With speeds above 1·5 × synchronism the losses in the coils short-circuited by the brushes rise to enormous values. At twice synchronous speed they are 4½ times, at 3 times that speed 32 times as large as with the corresponding series motor. With the latter class, taking the losses at synchronous speed as unity, those at twice the speed are double. To understand fully this important difference in the behaviour of the two classes of machines, we have to bear in mind that with the series motor, with rising speed, the field N becomes weaker and the circulating currents fall. With the repulsion motor the lines of the transformer flux N_2 are "cut," which is very useful at synchronous speed, as we have seen, but causes very considerable current to flow when the speed goes up, N_2 remaining nearly constant.

In the chapter on series motors we saw that the circulating currents had a good effect in so far as they shifted the phase of the field, giving it a watt component, so that the power factor improved. With the repulsion motor the effect is not quite the same, as the circulating currents are not induced altogether through the transformer action of N_1 but through "cutting" N_2 mechanically—*i.e.*, through exercising torque on the shaft. This mechanical energy is not altogether transformed into heat, developed in the short-circuited coils, but partly given back to the primary circuit in the form of electricity, transmitted through the mutual induction between the short-circuited coil and the primary field winding. In the way that we considered the magnetic condition we had two fields, one revolving backwards with the intensity $(B_1 - B_2)$ and one forward with the intensity $(B_1 + B_2)$. The armature coils, when running beyond synchronism, rotate more quickly than the latter field and therefore are giving energy back to the stator, thus giving the primary current a *negative watt component* in regard to the field voltage. Consequently the phase displacement between flux and field E.M.F. is reduced and the power-factor influenced unfavourably.

20.—Comparison of Series Motor and Repulsion Motor.

Summing up the points of difference between a repulsion and series motor we may say :—

1. The speed and power factor characteristic of the repulsion motor is the same as that of the series motor, except for the reaction of the circulating currents in the coils under commutation.

2. The copper loss in the armature and the core loss in the stator are higher, and the core loss in the rotor and the losses in the connections between armature and commutator are, in the proximity of synchronism, lower than that of the correspond-

ing series motor, the total losses being as a rule higher with the repulsion motor. If machines of both classes are built in such a way that they have equal losses the repulsion motor will be the heavier machine.

3. The repulsion motor offers certain advantages as to speed regulation and reversing which can be effected by brush shifting.

4. The armature winding of the repulsion motor can be one standard winding chosen to give good commutation regardless of voltage and frequency of the supply. Generally the winding will be wound for such a low voltage that no resistance connections in the armature are necessary, the carbon brush resistance being sufficient to keep down the circulating currents to a small amount (*see* our treatment of the subject on p. 50). This can be done the more easily as at synchronous speed the losses through circulating currents disappear automatically.

CHAPTER IV.

THE REPULSION MOTOR OF LATOUR-WINTER-EICHBERG.

21.—The Armature Winding as an Exciting Winding.

In the previous chapter we saw how the armature winding itself could serve as the secondary winding of a transformer, the primary being on the stator; as a matter of fact, the magnetising current i_m itself, producing the transformer flux, has its seat in the armature winding. The armature as an exciting winding can be utilised still further by omitting the winding W_1 of the repulsion motor (Fig. 117) and creating the flux N_1 by a current in the armature (*see* Figs. 118 and 119). This current naturally cannot in any case be conducted to the armature through the short-circuited brushes, because these brushes have such a position that only a transformation flux (N_2), not a dynamic flux, can be reproduced. N_1 must keep its position at right angles to the old brush line 2—2, and therefore

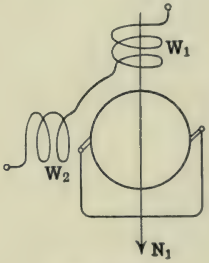


FIG. 117.

a second pair of brushes 1–1 (Fig. 120) with its line at right angles to the first must be used to conduct the exciting current to the armature. Fig. 114 is a diagram of the Winter-Eichberg, or Latour, motor, briefly the L.W.E. motor. In our new motor the stator carries only a transformer winding to transmit energy to the rotor, exactly as with the original repulsion motor, the only alteration being that the armature winding is the exciting winding. At first sight we may predict that the general character of such a machine is not very different from that of the repulsion motor proper, or series motor, hardly anything being materially changed. What strikes us first of all as the effect of transferring the seat of the exciting current into the armature itself is that the leakage flux must disappear, a flux

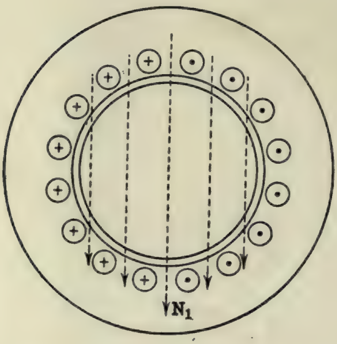


Fig. 118.—Production of Flux by Excitation from Stator.

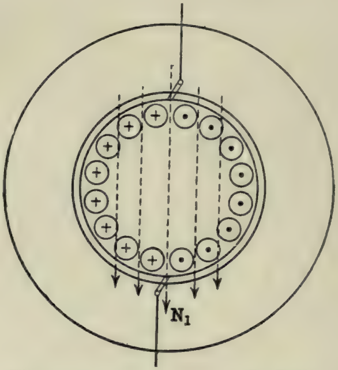


Fig. 119.—Production of Flux by Excitation from Rotor.

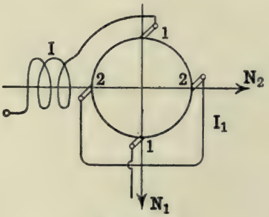


Fig. 120.

amounting to 10 to 15 per cent. of the total when the excitation is on the field separated from the armature by an air-gap.

The second question to ask would be: What happens if two currents flow through the same armature winding, the brushes being displaced 90 deg. from one another? Calling (Fig. 121) I the current entering the armature through the brushes 1—1 and I_1 the current conducted by 2—2, branches *b*—*c* and *a*—*d* will carry the current $I_1/2+I/2$, the branches *a*—*b* and *d*—*c* the current $I_1/2-I/2$, as can be read off directly from Fig. 121. Calling *r* the resistance of one-quarter of the armature and also the resistance of the total armature measured between 1—1 and 2—2, we find the copper losses to be

$$2\times\left(\frac{I_1}{2}+\frac{I}{2}\right)^2\times r+2\left(\frac{I_1}{2}-\frac{I}{2}\right)^2\times r=I^2\,.\,r+I_1^2\,.\,r,$$

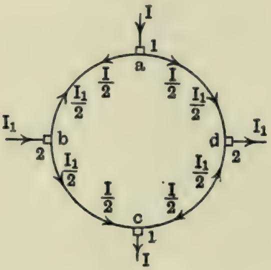


FIG. 121.

i.e., the copper loss in the armature is just as large as if the exciting current was flowing through the armature regardless of a current being conducted also by the brushes 2—2. In the ordinary way, if a resistance, *r*, carries two currents, I_1 and I, the losses would be larger, for $I_1=I$ double as large as if I_1 and I were flowing each by itself.

Thus we can produce the field in the proposed fashion without extra loss, except that caused by the drop in the additional brushes and their contact resistance, the advantage being that the leakage is smaller. A disadvantage, or rather a peculiarity, is that the losses in the armature are larger still than with the repulsion motor, the excitation losses taking place there instead of in the field. Therefore *the armature of the Winter-Eichberg motor will, as a rule, have a larger diameter than that of the series motor or ordinary repulsion motor* in order to accommodate the extra ampere-turns on its circumference. This is naturally compensated by the dropping out of the exciting ampere-turns from the stator; more copper losses in the rotor, less in the stator. With the ordinary repulsion motor we had—compared with the series motor—more copper losses in the rotor, but not less in the stator.

Special conditions are created by the field-turns as well as the armature-turns being fixed; in fact, both being identical. In the motors we have dealt with, the number of field-turns was smaller than that of the armature-turns and, due to our now being tied as to the high number of field-turns, a serious disadvantage would certainly be the consequence. Now we have only to remember that we are dealing with alternating current, which can be so easily transformed, and a transformer, T (Fig. 122), may be introduced, the ratio of transformation of which we are at liberty to make suitable to the particular case. In this way we may regain flexibility. Very often T is arranged so that the ratio of transformation can be regulated by altering the number of turns of the secondary coil.

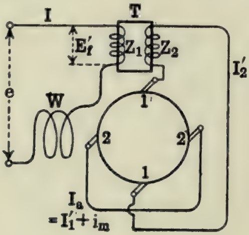


FIG. 122.

22.—The Compensation.

We might have dismissed the Latour-Winter-Eichberg motor as a special case of the repulsion motor, having no particularly new features, if there were not a new element introduced through the brushes 1—1 being shifted 90 deg. from the direction of the flux N_2. We know that under these conditions the transformation flux N_2 induces an E.M.F. (E_2) in the armature winding through the latter cutting its lines and that a torque *may* be exercised by this flux N_2 on the armature conductors carrying the current I. To start with, we can say that the torque between N_2 and I will be zero, because I and N_1 are in phase (except for hysteretic phase displacement) and the phase of N_2 is 90 deg. behind N_1, and consequently also I and N_2 have a phase displacement of 90 deg. Naturally, if through hysteresis lag and other reasons, the angle between I and N_2 should be altered they would be able to act on one another. Disregarding this, the torque will be zero. The E.M.F. E_2 induced in the armature winding in the direction 1—1 through cutting the lines of N_2—as it were, an E.M.F. induced in the "exciter circuit," the brushes 1—1 representing only its "terminals"—is in phase with N_2 (Fig. 123) and therefore E_2 is displaced 90 deg. from I. Further we may say that as N_2 is lagging behind N_1 and I, E_2 is also lagging behind I. Now we have two E.M.F.s. in our exciter circuit 1—1 (Fig. 123), the E.M.F. of self-induction E_f (or generally E_L, if we include armature leakage) caused by the alternation of N_1 which was *leading* in respect to the current I by 90 deg. and E_2 *lagging*. Thus E_2 tends to compensate E_f. We may say: *Through the*

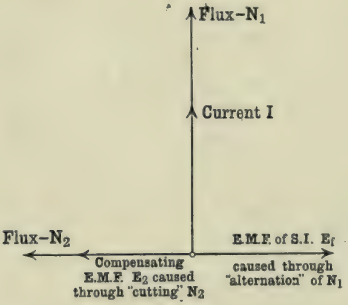

Flux–N_1
Current I
Flux–N_2
E.M.F. of S.I. E_f
Compensating E.M.F. E_2 caused through "cutting" N_2
caused through "alternation" of N_1

FIG. 123.

armature being used as an exciting winding in the L.W.E. motor, besides the harmful E.M.F. of self-induction, a new E.M.F. is induced in the exciter circuit, which tends to reduce the former. This E.M.F., E_2, may be called the "compensating E.M.F."

When the motor is running at synchronous speed, the two fields N_1 and N_2 have equal intensity, and the speed of cutting the lines is the same as the speed of alternation, so that at synchronism, E_2 is equal to E_f and compensates it entirely. Consequently there is no wattless E.M.F. necessary to produce a field just as with the field of a direct current motor. *At synchronism the power factor of the L.W.E. motor is practically equal to unity.*

23.—E.M.F.s, Currents and Fluxes in the L.W.E. Motor.

Fig. 118 shows a diagram of the series or ordinary repulsion motor and Fig. 119 that of the L.W.E. motor in their simplest forms, the only difference being the E.M.F., E_2, which reduces the angle of phase displacement ϕ. The diagram of the secondary (rotor) circuit 2—2 of the two repulsion type motors has been drawn separately (Fig. 125A), Figs. 124 and 125 referring to the primary or main circuit only. When $N_1 = N_2$ (at synchronism) the magnetising current for N_1, I'_2 (Fig. 116) becomes equal to the magnetising current for N_2, i_m, the number of turns for both being the same. Generally

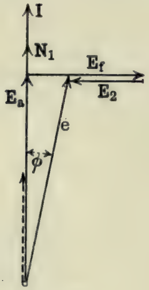


FIG. 125.

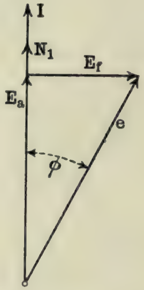


FIG. 124.

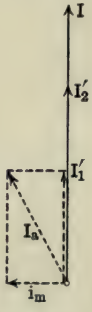


FIG. 125A.

$$\frac{I'_2}{i_m} = \frac{N_1}{N_2},$$

and as $\frac{N_1}{N_2}=\frac{n}{n}$, where n_s equals synchronous speed and n the actual speed,

$$\frac{I'_2}{i}=\frac{n}{n}.$$

It is interesting to note that the two magnetising currents have such a simple relation one to another, just as the two fields.

The relations between the other currents are fixed by the ratio of transformation of the two transformers—*i.e.*, that in the motor with the winding W (Fig. 122) as primary, $q_1=\frac{z_a}{z_w}$ and that in the exciter-transformer T, $q_2=\frac{z_2}{z_1}$.

Then $$I'_1=\frac{I}{q_1}\ ;\ I'_2=\frac{I}{q_2}\ ;\ I'_2=I'_1\,.\,\frac{q_2}{q_1}.$$

The field ampere-turns producing N_1 are I'_2 × armature-turns, the "working armature ampere-turns" (rotor ampere-turns minus magnetising ampere-turns) are I'_1 × armature-turns; therefore the ratio, field ampere-turns to working armature ampere-turns is equal to the ratio of transformation of the exciter transformer divided by that of the motor transformer.

It may appear at first sight as if, on account of its compensating qualities, the L.W.E. motor might be designed regardless of the power factor, and that consequently the ratio, field ampere-turns to working armature ampere-turns, might be very high, the air-gap being as large as with a direct current machine. However we are not so entirely free in our design because with a very high ratio of field ampere-turns to working armature ampere-turns the magnetising currents i_m and I'_2 having to produce N_2 and N_1 will be very large, and will become prohibitive with air-gaps approaching those of direct-current machines. With the L.W.E. motor the armature copper losses are already higher than desirable, the armature having to carry the exciting currents for both fields N_1 and N_2 as well as the working current I'_1. If the ratio, field ampere-turns to working armature ampere-turns, were 1, I'_2 as well as i_m would at synchronism be equal to the working armature current I'_1.

Consequently, flux and body of the motor being the same, the ampere-turns in the armature would be three times as much as with the plain series motor, and one and a-half times as much as with the plain repulsion motor. It would be hardly possible to provide enough copper section for this enormous number of ampere-turns. Consequently, the air-gap, and with it the magnetising ampere-turns, must be kept as small as possible with all alternating-current commutator motors for the following reasons :—

With the plain series motor, to improve the power factor ;

With the plain repulsion motor, to improve the power factor and to reduce the armature copper losses ;

With the L.W.E. motor, to reduce the armature copper losses.

The ratio of ampere-turns must be kept small, also with regard to the overload capacity and the starting conditions of the motor, and the fact that only at synchronous speed perfect compensation takes place.

As a rule the ratio $\frac{\text{magnetising ampere-turns}}{\text{working armature ampere-turns}}$ will not exceed the figure 0·50 in any case. With the L.W.E. motor we have then $\frac{q_2}{q_1}=0{\cdot}5$, $\frac{I'_2}{I'_1}=0{\cdot}5$, and at synchronism when $i_m=I'_2$ also $\frac{i_m}{I'_1}=0{\cdot}5$. The resultant current $I'_a=\sqrt{I_1'^2+i_m^2}=1{\cdot}12\times I'_1$ $=2{\cdot}24\times I'_2$. The exciting current I'_2 flowing through the brushes 1—1 is therefore less than half the current flowing through the pair 2—2 (Fig. 122). Consequently the number of exciter brushes can be kept smaller. This is usually done in that way ; that, for instance, with a four-pole motor four brush arms are provided for the working current (brushes 2—2), but only two brush arms for the exciter brushes 1—1, opposite segments being connected together.

A few interesting relations between the voltages E_a, E_f and E_2 (Fig. 125) may be developed.

We can put :

$$E_a=\text{constant}\times n\times N_1\times\frac{1}{q_1}=\text{constant}\times n_s\times N_2\times\frac{1}{q_1}, \text{ and } \frac{N_2}{N_1}=\frac{n}{n_s}.$$

Further: $$E_f = \text{constant} \times n_s \times N_1 \times \frac{1}{q_2},$$

$$E_2 = \text{constant} \times n \times N_2 \times \frac{1}{q_2}.$$

Consequently $$\frac{E_2}{E_f} = \frac{n}{n_s} \cdot \frac{N_2}{N_1} = \left(\frac{n}{n_s}\right)^2.$$

The ratio between compensating E.M.F. and E.M.F. of self-induction in the exciter circuit is equal to the square of the ratio $\dfrac{\textit{actual speed}}{\textit{synchronous speed}}$. At synchronous speed $\dfrac{E_2}{E_f} = 1$, as we have already found from general considerations. *We see that* E_2 *changes very quickly with the speed*, and that at, say, 25 per cent. below synchronism

$$\frac{E_2}{E_f} = 0{\cdot}75^2 = 0{\cdot}56,$$

so that $$E_f - E_2 = 0{\cdot}44 \times E_f,$$

only 56 per cent. of the E.M.F. of self-induction is compensated.

At twice synchronous speed $\dfrac{E_2}{E_f} = 4$; $E_f - E_2 = -3 \times E_f$. The motor would be very much over-compensated, and the power factor become very small.

Naturally E_f only represents the E.M.F. of self-induction in the exciter circuit, not including leakage of any sort. Complete compensation will therefore take place at a speed slightly over synchronism. The latter speed may artificially be raised to a certain degree by inserting induction coils in the exciter circuit.

Other interesting relations are

$$\frac{E_2}{E_a} = \frac{n}{n_s} \cdot \frac{q_1}{q_2}; \quad \frac{E_f}{E_a} = \frac{n_s}{n} \cdot \frac{q_1}{q_2}; \quad E_2 = E_a \cdot \frac{E_a}{E_f} \cdot \left(\frac{q_1}{q_2}\right)^2.$$

At synchronism $$\frac{E_2}{E_a} = \frac{E_f}{E_a} = \frac{q_1}{q_2}.$$

24.—The Circle Diagram of the L.W.E. Motor.

Through the presence of E_2 the circle diagram will have to be modified for the L.W.E. motor. Fig. 126 shows the circle diagram for a motor where the influence of the resistances and core losses has not been taken account of.

We make $OA = e =$ terminal voltage; draw the circle with OA as diameter, as in the previous diagrams. Again OB is the useful E.M.F., E_a; AB is the wattless E.M.F., now no longer E_f, but $AB = E_f - E_2$, or rather $E_L - E_2$, if the whole self-induction in the motor is combined.

In order to find E_f itself, which as we know gives us a measure for the current I, we draw AC vertical to OA and make $AC = \frac{q_1}{q_2} \times OA$. The ratio $\frac{q_1}{q_2}$ may have been found experimentally or by calculation. Draw OC, find D by making $OD = DC$, and DF vertical on OC. Where DF cuts a line GF 45 deg. inclined to OA—*i.e.*, at the point F—is the centre of a second circle which we draw through O and C. Producing the vector $AB = E_f - E_2$, so that it cuts the new circle in H, we have $AH = E_f$ *and* $BH = E_2$, AH *being at the same time a measure of the current.* This diagram is approximate only, as actually the locus for H is not a circle. For very high speeds it must not be used at all. We note that not the "stand still" or "short-circuit" current AO is the largest possible, as in the previous diagrams, but the current AK, where AK passes through F. This is due to the choking effect of the field (E_f) being reduced by E_2 which grows as the speed increases, E_2 subtracting directly from E_f, whilst the counter E.M.F. E_a is only at right angles to it. At synchronous speed the current is $AC = \frac{q_1}{q_2} \times AO$, or $\frac{q_1}{q_2}$ (of the order 0·5) times the short-circuit

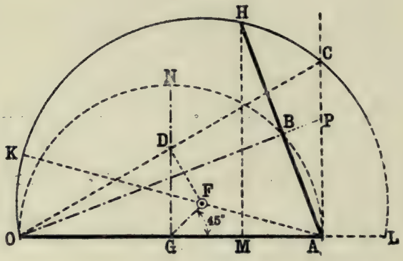


FIG. 126.

current. Then the power factor OB (*see* previous circle diagrams) is unity.

Above synchronism the current becomes a leading one, and at the same time falls off more and more until at very high speeds it approaches zero (in the diagram the value AL), and *leads* in respect to the terminal voltage *by* 90 *deg.* The watt-component of the current—*i.e.*, the vertical line HM—is, just as in our previous diagrams, a measure of the output of the machine. *Its maximum is reached if* HM *passes through* F. With the series motor the maximum output was GN, on exactly the same scale, and was considerably less. Thus, with the same short-circuit current—*i.e.*, the same ratio of field ampere-turns to armature ampere-turns—the overload capacity of the L.W.E. motor is larger than that of the series motor. But with all motors equally, series, repulsion and L.W.E. motors, the short-circuit current ought to be as large as possible, to give good *starting conditions*, there being practically no difference whatever in the behaviour at starting, the faculty of compensating appearing with the L.W.E. motor only when running.

It is not desirable to make our diagram more complicated by introducing more lines, for instance, for the speed or the rotor current. The speed as a percentage of the synchronous speed can be easily found by taking account of the equations

$$n = n_s \times \frac{q_1}{q_2} \times \frac{\mathrm{E}_{\prime\prime}}{\mathrm{E}_f} = n_s \times \frac{q_1}{q_2} \times \frac{\mathrm{OB}}{\mathrm{AH}},$$

$$\text{or} \quad n = n_s \times \frac{q_2}{q_1} \times \frac{\mathrm{E}_2}{\mathrm{E}_f} = n_s \times \frac{q_2}{q_1} \times \frac{\mathrm{HB}}{\mathrm{OB}} = n_s \times \frac{q_2}{q_1} \times \tan \mathrm{HOB},$$

$$\text{or} \quad n = n_s \times \sqrt{\frac{\mathrm{E}_2}{\mathrm{E}_f}} = n_s \times \sqrt{\frac{\mathrm{HB}}{\mathrm{AH}}}.$$

We may calculate the speed from the amount AP cut off by OB from the vertical AC; where $\mathrm{AP} = \mathrm{OA} \times \frac{q_1}{q_2} \times \left(\frac{n_s}{n} - \frac{n}{n_s}\right)$, so that AP is actually only a measure of $\left(\frac{n_s}{n} - \frac{n}{n_s}\right)$, from which $\frac{n}{n_s}$ must be calculated. When AP becomes identical with AC, the speed is 0·618 of the synchronous speed. It is necessary to start from the dimension AP if it is desired to find the current, power and power factor when the speed is given.

The rotor current passing through the brushes 1—1 naturally is $I'_1 = AH$. The working component in the portion of the rotor short-circuited by means of the brushes 2—2 is $I'_2 = I'_1 \times \frac{q_1}{q_2}$, whilst its magnetising component i_m can be found from the relation $\frac{\dot{i}_m}{I'_2} = \frac{n}{n_s}$.

The true locus of H can be found graphically or by calculation, considering that

$$E_f = AH = \frac{AB}{2} \pm \sqrt{\left(\frac{AB}{2}\right)^2 + \left(\frac{q_1}{q_2} . OB\right)^2}.$$

It is the object of the circle diagrams for the ideal motors to compare in principle an ordinary series motor with a repulsion motor of the Thomson or Latour-Winter-Eichberg type, the supposition being that the same iron body is used in all cases, that the normal fluxes are the same, the ratio field ampere-turns/working armature ampere-turns being assumed as 0·4 or 0·5 or on similar lines. *In dealing with a particular case it is, as already explained, preferable to construct a linear diagram for every point of the load,* taking account, as much as possible, of every detail, such as leakage and saturation, the latter making the attempt to construct a circle diagram more or less futile

SPECIAL TYPES.

If we had defined as normal types the plain series and the original repulsion motor, the motors with and without transformation of the armature energy, the L.W.E. motor ought to have been treated as a "special type," because nothing distinguishes it from the repulsion motor except the field being excited from the armature instead of the stator. The two pairs of brushes with a two-polar machine and the induction of the compensating E.M.F. in the exciter circuit, however, make it a separate class of machine with some new features.

CHAPTER V.

SPECIAL TYPES.

25.—Modifications of the Series Motor.

The *series motor* is so simple in itself that very few essential modifications are possible. The compensation of the armature reaction through short-circuiting the compensating winding has been dealt with (p. 38), as well as means to improve the power factor, by shifting the phase of the flux, and means to suppress sparking and circulating currents by special windings on the commutating poles (pp. 43 and 57).

The solution of the mechanical problems connected with the series motor has given rise to some special designs. The resistance connections of the armature are generally of high resistance metal (usually German silver) so as to obtain the necessary resistance with as short a length as possible, the resistances being placed in the slots themselves as indicated by Figs. 127, 128, 129. As an example of a different arrangement we may mention that of the Siemens-Schuckert Company (Richter) where the resistances are made of ordinary high conductivity copper, having for producing the resistance quite a considerable number of turns and being placed under the pole tips instead of the " neutral " so that the current flowing through it produces some torque (Fig. 130).

The Siemens-Schuckert Company have combined the main field winding, the compensating winding and the windings of the compensating poles into a uniform resultant winding with identical former-wound coils, the whole " splitting " off of the different portions being carried out by special connections. In this way some copper is saved and the copper loss reduced, the "resultant" winding containing less ampere-turns than the algebraical sum of its components.

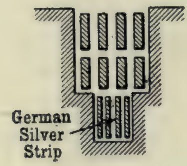


FIG. 127.

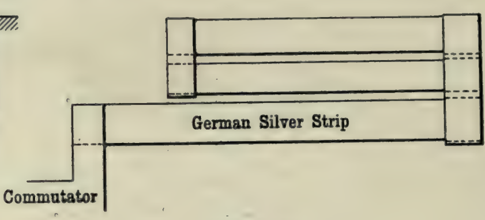


FIG. 128.

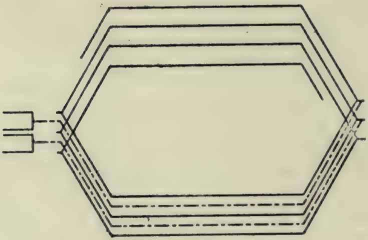


FIG. 129.

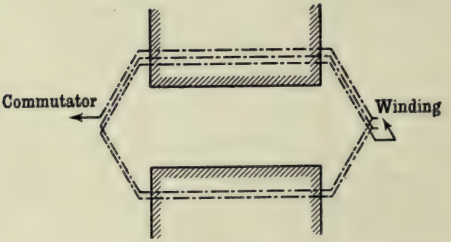


FIG. 130.

26.—Modifications of the Thomson and Latour-Winter-Eichberg Repulsion Motors.

The *repulsion motor* has been brought to great perfection by Messrs. Brown, Boveri & Co., who are applying an arrangement devised by Dery, the chief advantage of the repulsion motor, namely, speed regulation by brush shifting, having been developed specially.

The motor has two pairs of brushes, which are short-circuited in pairs, as shown in Fig. 131. Brushes 1 and 2 are fixed, whilst 3 and 4 can be shifted in order to effect speed regulation. The two pairs of brushes are equivalent to a single set 5—6. If α is the angle by which the brushes 3—4 have been shifted out of the pole line the effect is the same as if the resultant brush had only been moved by $\alpha/2$ deg., so that there is very much less sensitiveness as to brush position. With a small four-pole machine the whole of the regulation would have to be effected with the original repulsion motor by shifting the brushes over a quarter of the commutator circumference, *i.e.*, a few inches. With the Dery motor 180 deg. of the commutator are available for regulation. One further peculiarity resulting through the application of four brushes may be mentioned. Fig. 131 shows the diagram of a Dery motor, where the stator winding has been resolved in the exciter and transformer winding A_1 and A_2. It is interesting to see that the conductors between 1 and 3 as well as those between 2 and 4 do not carry any current. Therefore the armature winding becomes more effective as to its magnetising power, because the leads between 1—3 and 2—4 would have contributed to the resistance of the armature,but only a very little to the production of magnetism. (*see* our discussion in Section 4 on the Distributed Winding).

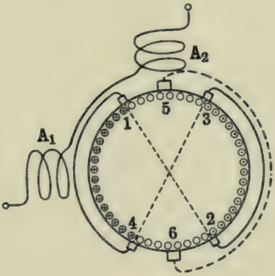


FIG. 131.

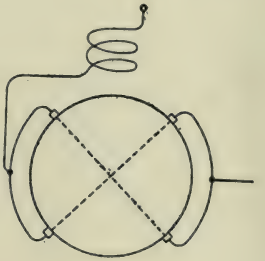


FIG. 132.

Other methods to improve the rotor winding in this respect have been suggested by Latour and Danielson (Fig. 132) who, with an ordinary series motor, apply four brushes for each pair of poles. The principle made use of is the same as that just developed. Here the angle between the brushes is 90 deg., so that they may be connected also in the L.W.E. fashion, in order to utilise the good qualities of this motor when running in the proximity of synchronism. As a disadvantage must be considered the broad neutral zone of 90 deg. as far as commutation and utilisation of armature winding is concerned; for the same reason great difficulties result when commutating poles are used.

In Chapter V. we pointed out the difficulties in building an alternating shunt motor which would run at constant speed. The problem did not admit of a practical solution because the field winding is an inductive resistance and therefore the exciting current i_f and the flux N lag nearly 90 deg. behind the terminal voltage. Now, we saw in the sections on the L.W.E. motor that the exciter circuit, placed on the armature in this case, has, *when revolving at synchronous speed*, no longer any inductive resistance, its E.M.F. of self-induction E_f being balanced by the compensating E.M.F., E_2. Therefore, if a L.W.E. has reached synchronism the exciter brushes 1—1 might be switched over on the main voltage; naturally, using a transformer as the exciter circuit necessitates only a very small voltage, somewhere near 10 per cent. of the exciting voltage necessary without compensation (Fig. 133).

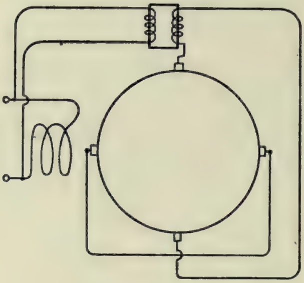


FIG. 133.

Such a motor must be started as a series motor. The small exciter transformer may be embodied in the motor itself by providing co-axial with the winding W (Fig. 134) a small winding *w* of a few turns, which are connected to the brushes 1—1 (Fynn's motor). In order to facilitate the switching over from the starting to the running arrangement, a scheme has been devised by Osnos which allows this to be effected by simply closing a single-pole switch. This method (Fig. 135) is being used by Messrs. Lahmeyer. As the starting connection is not interrupted, when *w* is switched on, the exciter circuit 1—1 is excited both by the main current and by the current derived from *w*. Should the speed rise, the main current dropping, the motor cannot race because there is always the supply of current by *w*.

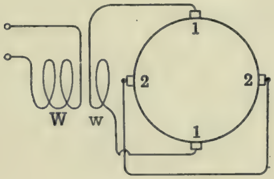


Fig. 134.

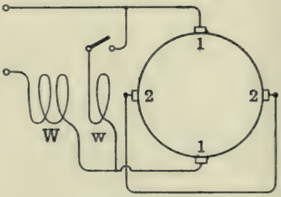


Fig. 135.

Arnold and Lacour have devised a method to regulate, to a certain degree, the speed of such a "shunt type" motor. They not only excite the machine from the rotor, but also through a stationary winding connected in series with the rotor exciter circuit (brushes 1 – 1) and co-axial with the latter. Both circuits may be connected together either so that their ampere-turns add or substract. In this fashion the field N_1 and with it the E.M.F. of self-induction of the field circuit (E_f) (*see* Fig. 125) may be altered, a new speed where the compensating E.M.F., $E_2 = E_f$ resulting. By employing several steps on the stator exciting winding, or by means of induction coils (*see* p. 117), a finer grading of the speed is possible.

CHAPTER VI.

EXAMPLES AND PERFORMANCES OF COMMUTATOR MOTORS.

27.—Series Motors.

The series motor of Messrs. *Siemens* has the following features (*see* also pp. 59 and 121) :—

Stator of " Induction motor."

Commutating poles with shunt and series winding.

Combination of all stator coils into one resulting winding with former-wound coils.

Resistance connectors between armature winding and commutator consisting of copper and have several turns, forming a proper coil.

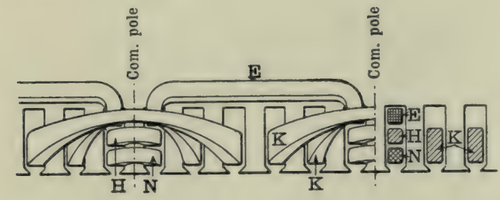


Fig. 136.

Fig. 136 shows diagrammatically and Fig. 138 the photograph of a stator with the older type of winding where exciting winding (E), compound winding (K), shunt and series coils on the commutating poles (N and H) are each wound separately. Fig. 137 is the diagram of connections, where A=armature and T the main transformer.

A stator with former-wound coils is shown in Fig. 139, Fig. 140 being the diagram of connections belonging to it. These

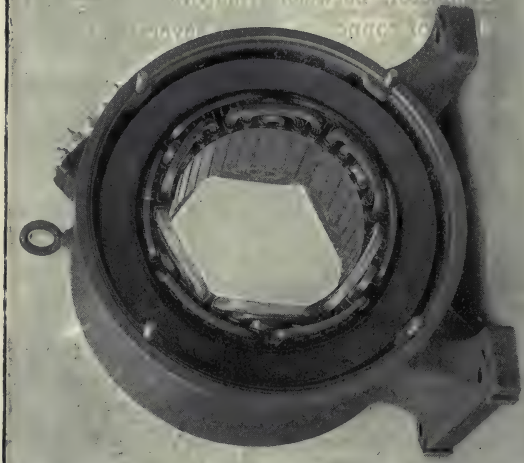


Fig. 138.

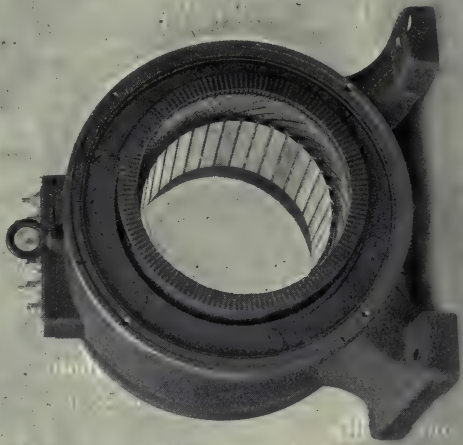


Fig. 139.

Siemens Motors.

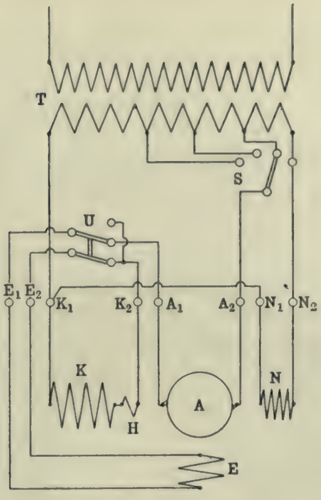


FIG. 137.

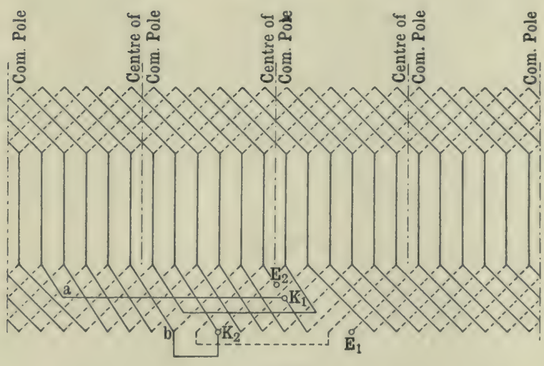


FIG. 140.

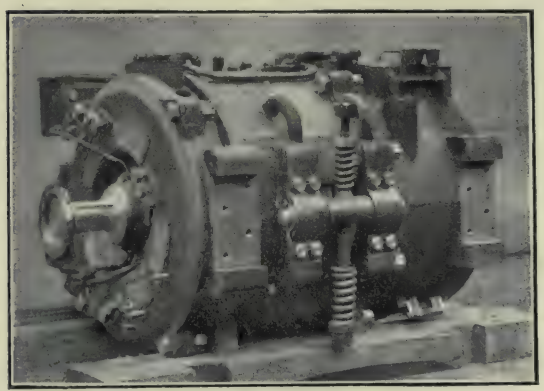


FIG. 143.—SIEMENS MOTOR.

illustrations refer to small machines, whilst in Fig. 141 the stator of a machine for 175 H.P., 700 revs. per min. and 25 periods is reproduced. Fig. 142 shows the armature and Fig. 143 a view of the complete motor.

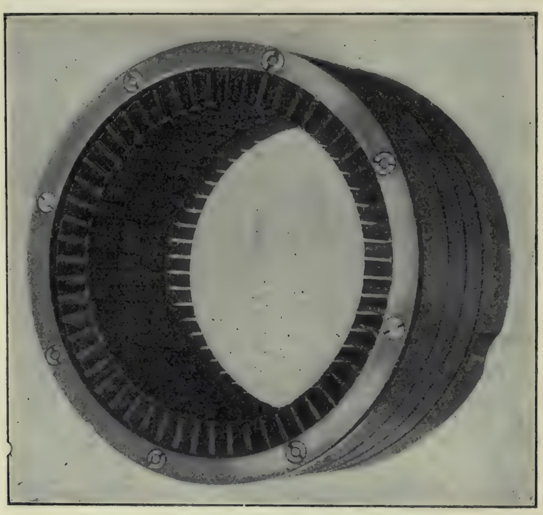


FIG. 141.

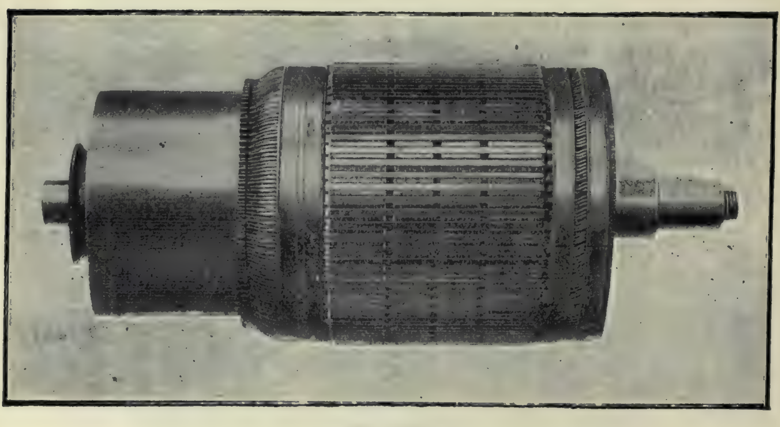


FIG. 142.

The *Oerlikon* motor :—

Stator of " Induction motor " (closed slots).

Commutating poles with only one winding (H) carrying the main current.

Branches in parallel with H are formed in order to alter the phase of the current in H so that it tends to reduce the circulating currents (p. 59), either by means of resistance (W), Fig. 144, or various connections, as shown in Figs. 145 to 147.

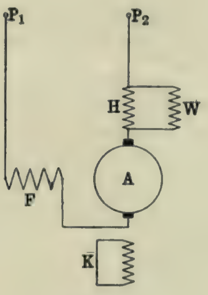


Fig. 144.

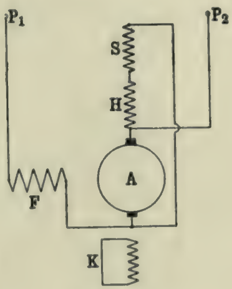


Fig. 145.

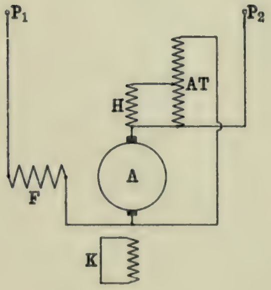


Fig. 146.

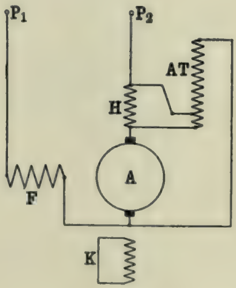


Fig. 147.

S = Choking coil. AT = Auto-transformer.

The compensating winding (K) is short-circuited in itself.

No resistance strips in the armature.

Figs. 148 to 153 show sectional drawings and photographs of a six-polar railway motor for 225-250 volts, 60 h.p. and about 750 revs. per min. The motor, including gear, has a weight of 3,700 lb. The gear ratio is 1 : 5·15. The motor may be used

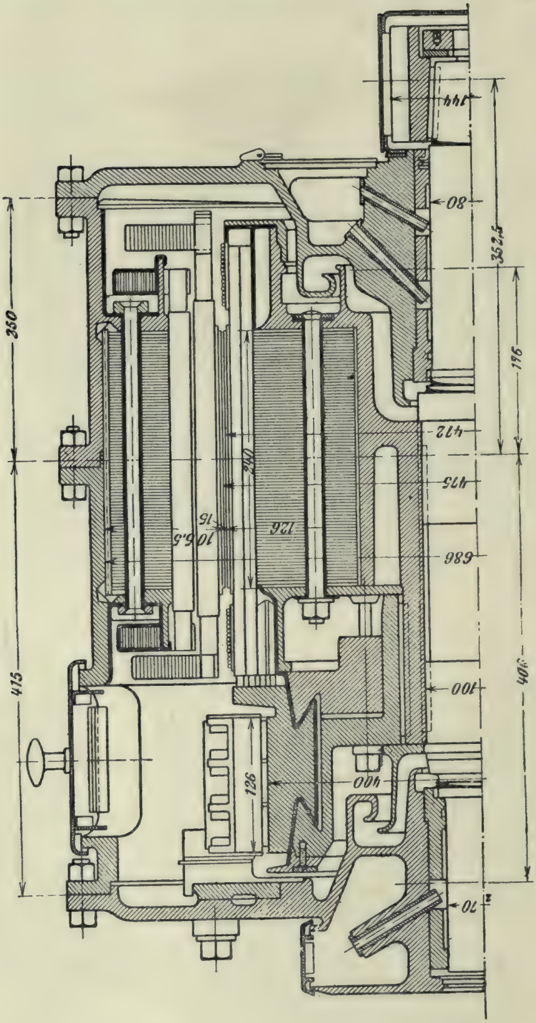


Fig. 148.—60 h.p. Oerlikon Motor.

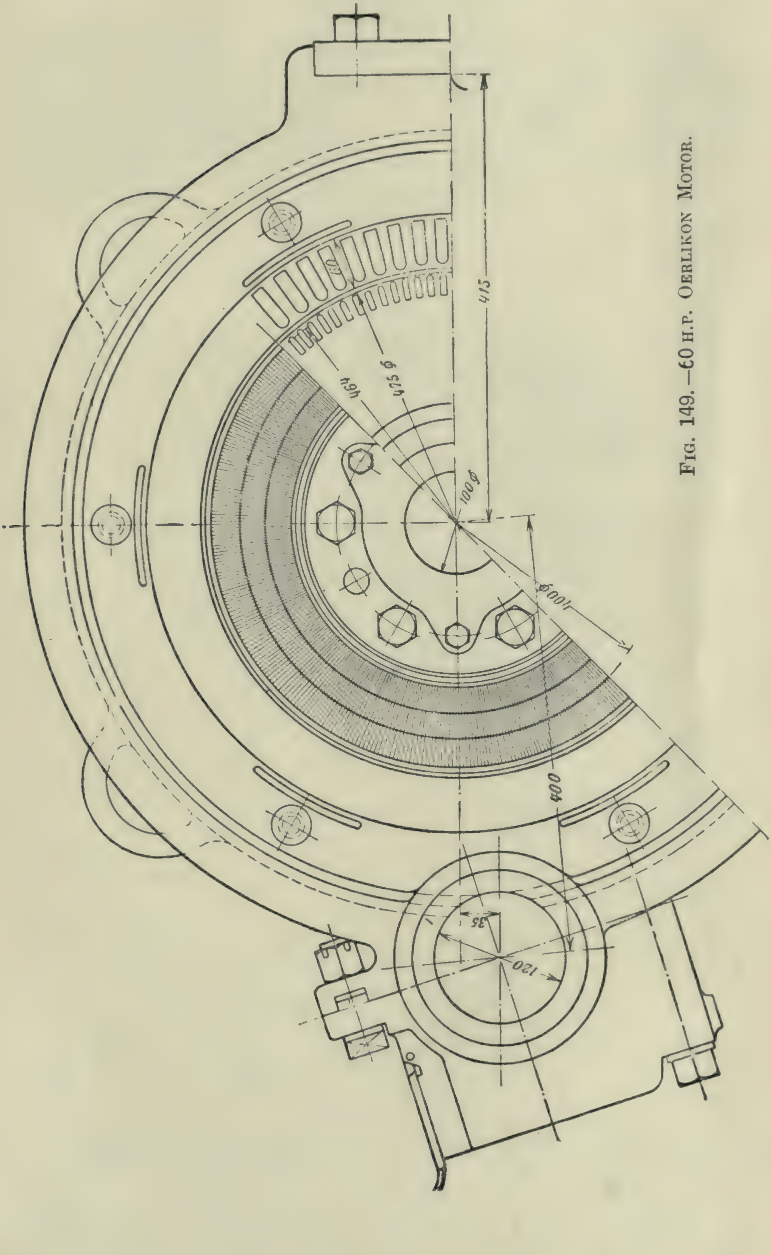


FIG. 149.—60 H.P. OERLIKON MOTOR.

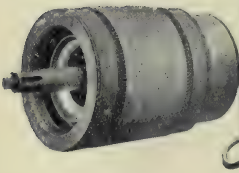


FIG. 150.

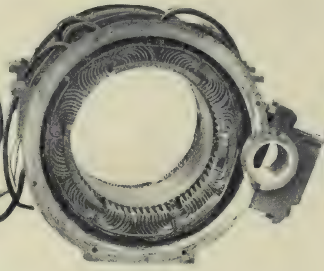


FIG. 151.

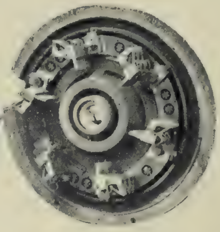


FIG. 152.

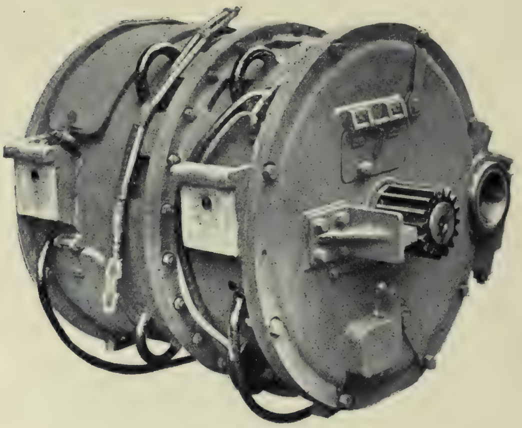


FIG. 153.

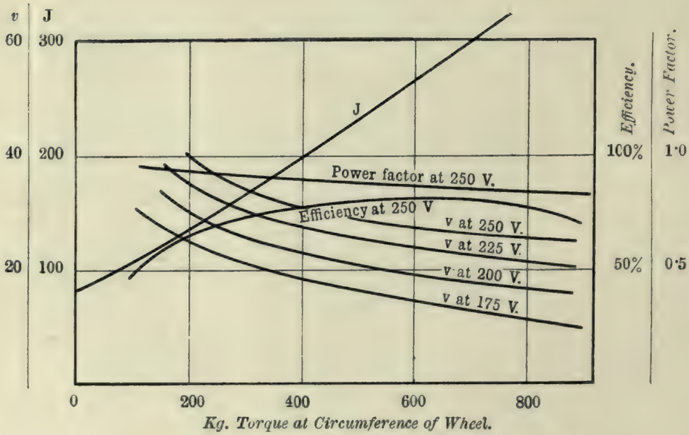


FIG. 154.—60 H.P. OERLIKON MOTOR.

v = Current in amperes.
J = Speed in kilometers per hour at circumference of wheel.

for direct current also, the compensating winding being used in series with the main field winding for excitation, and the voltage being reduced about 25 volts. Performance curves for starting and running with alternating and direct current are plotted in Figs. 154 to 156.

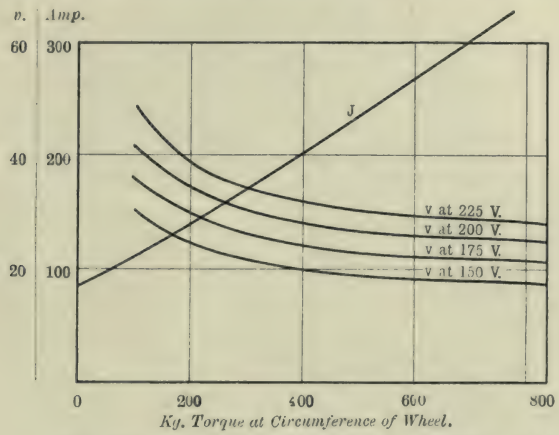


FIG. 155.—60 H.P. OERLIKON MOTOR. (*See* Fig. 154.)

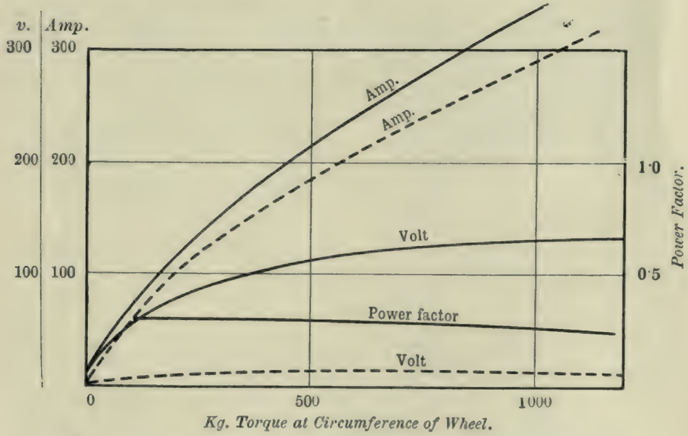


FIG. 156.—STARTING TORQUE OF 60 H.P. OERLIKON MOTOR.

——— With alternating current·
- - - With direct current.

Fig. 157 shows a 200 H.P. motor for 15 periods, 220 volts, 650 revs. per min., the weight of the motor without gear being about 7,000 lb. Fig. 158 is the performance curve of this motor for constant current. Curve I is that of the voltage.

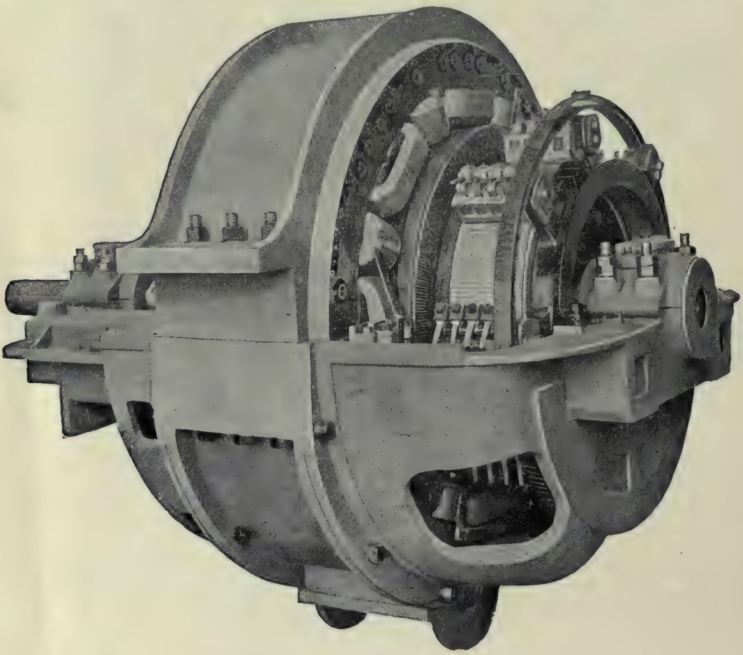


FIG. 157.—200 H.P. OERLIKON MOTOR.

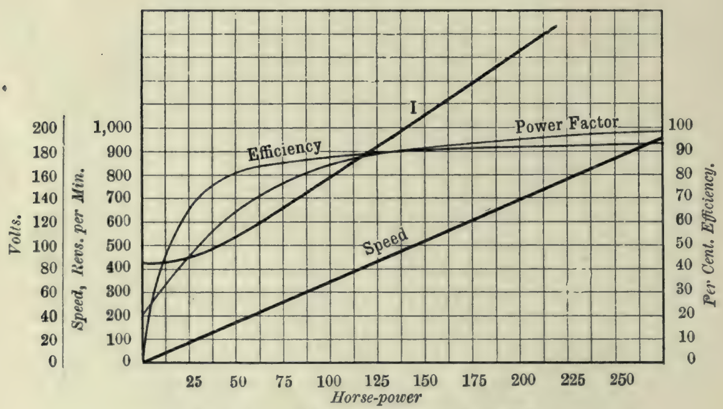


FIG. 158.—200 H.P. OERLIKON MOTOR.

Messrs. *Kolben*, in Prague, have built the six-polar motor shown in Fig. 159. It is for 10 H.P., 220 volts, 50 periods, 1,000 revs. per min.

The commutating poles have shunt winding only, which is wound so that it can be connected direct to the supply voltage. The armature is series wound and has resistance connectors consisting of thin copper wire of several turns wound bifilar (not proper coils as with the Siemens motor) and placed in the slots on the top of the armature winding.

When used as an alternating current motor the efficiency is 65 per cent., the power factor 85 per cent. When used as a direct current motor, the efficiency is 71 per cent.

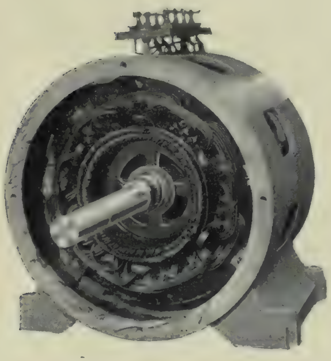


FIG. 159.—10 H.P. KOLBEN MOTOR.

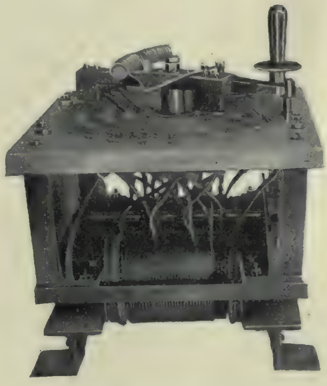


FIG. 160.—AUTO-TRANSFORMER.

The exciting winding is arranged in two parts, which are usually in parallel, but can be connected in series if it is desired to run the motor with direct current.

Fig. 160 illustrates the auto-transformer used for starting the motor.

The firm who were the first on the market with alternating commutator motors is the *Westinghouse Company*. Their motor (type No. 132) is represented by Figs. 161 to 164.

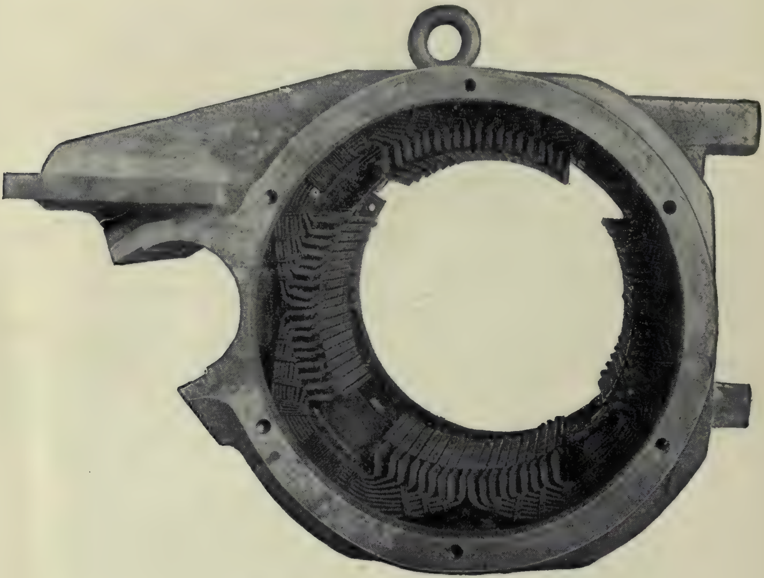


Fig. 161.—Westinghouse Motor.

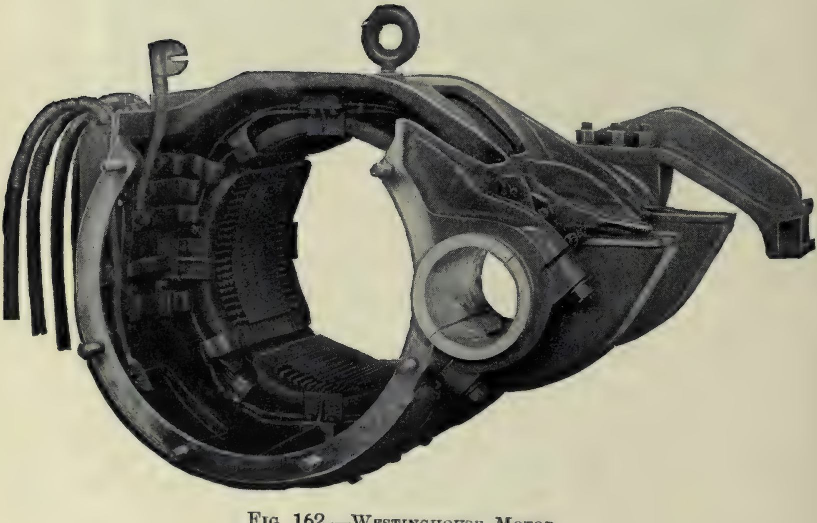


Fig. 162.—Westinghouse Motor.

The motors have "concentrated" field coils (p. 15), no commutating poles being provided. The connections between armature winding and commutator consist of German silver.

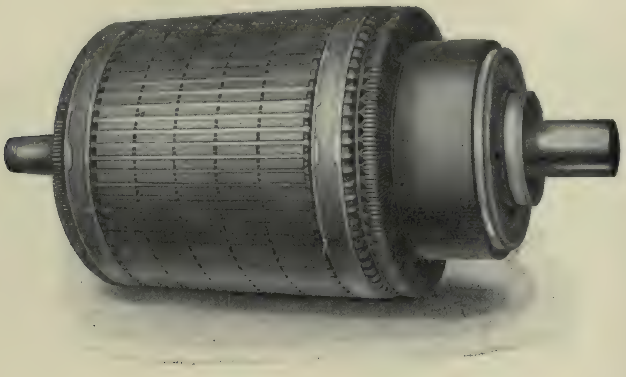


Fig. 163.

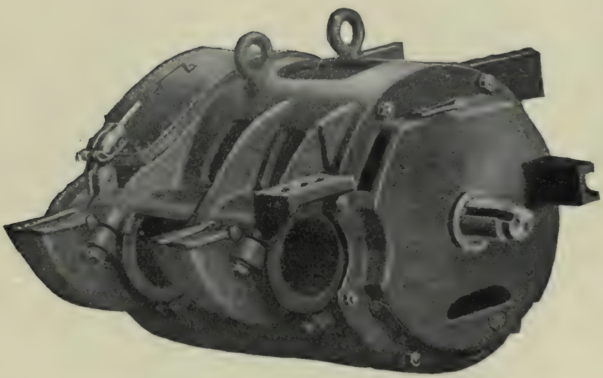


Fig. 164.

Fig. 165 is the performance curve of a 100 H.P. motor for 225 volts, 25 periods. The weight of this motor is 5,200 lb., including gear and gear case.

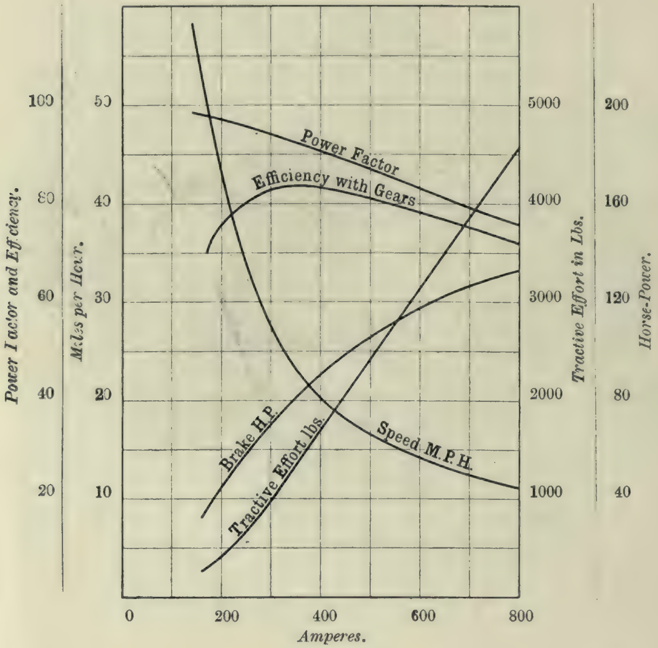


Fig. 165.

225 Volts. 25 Periods. 18 : 65 Gear Ratio. 36 in. Wheels.

The machines of the *General Electric Co.* are built on similar principles to those of the Westinghouse Company. Their motor GEA 605 A is shown in Figs. 166 and 167. It has four poles,

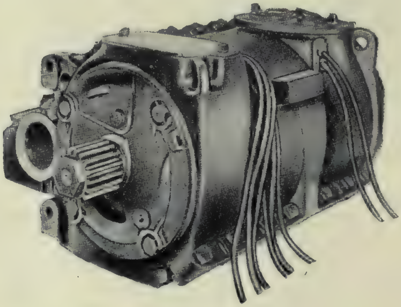


Fig. 166.

and is rated at 75 H.P., 250 volts, 25 periods, 750 revs. per min. The weight of the motor is 4,200 lb. net, that of the armature alone 1,200 lb. Performance curves for this machine when supplied with alternating current and direct current are given in Figs. 168 and 169. In the latter case the field coils are all connected in series, ordinarily they are in parallel,

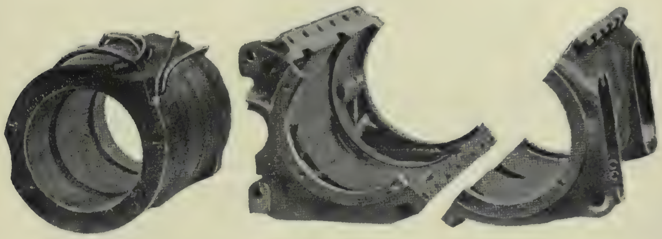


FIG. 167.

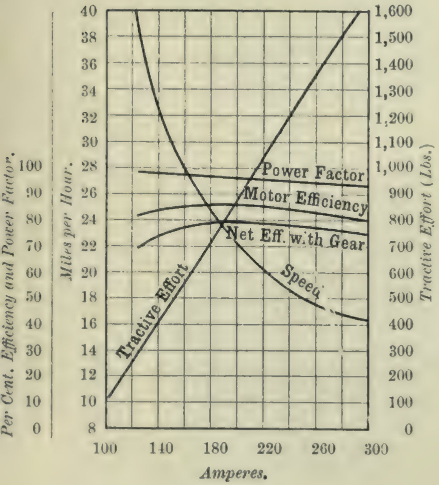


FIG. 168.

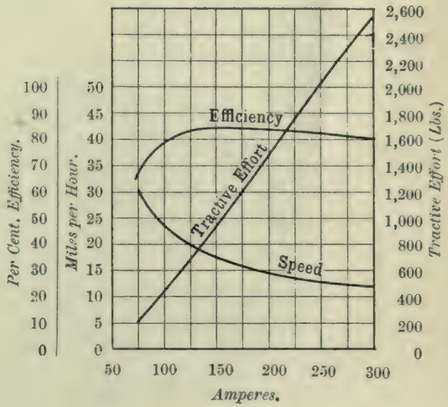


FIG. 169.

28.—Repulsion Type Motor.

Motors of the Winter-Eichberg type are manufactured by the *Allgemeine Elektrizitäts Gesellschaft* in Berlin.

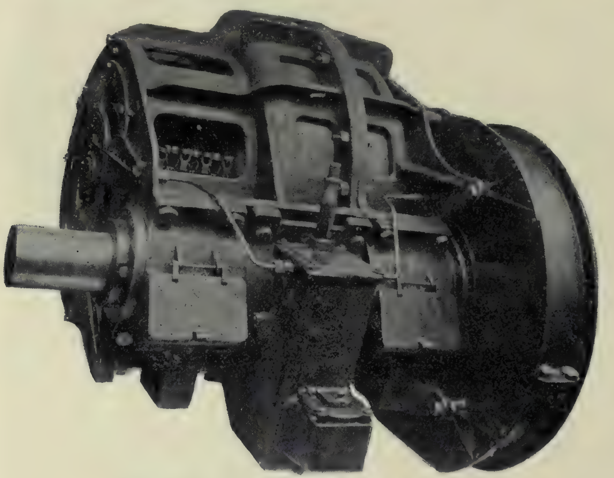


Fig. 170.—WE Motor of the A.E.G.

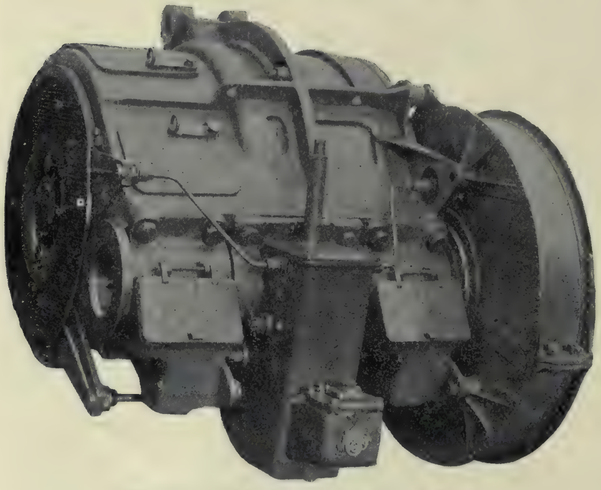
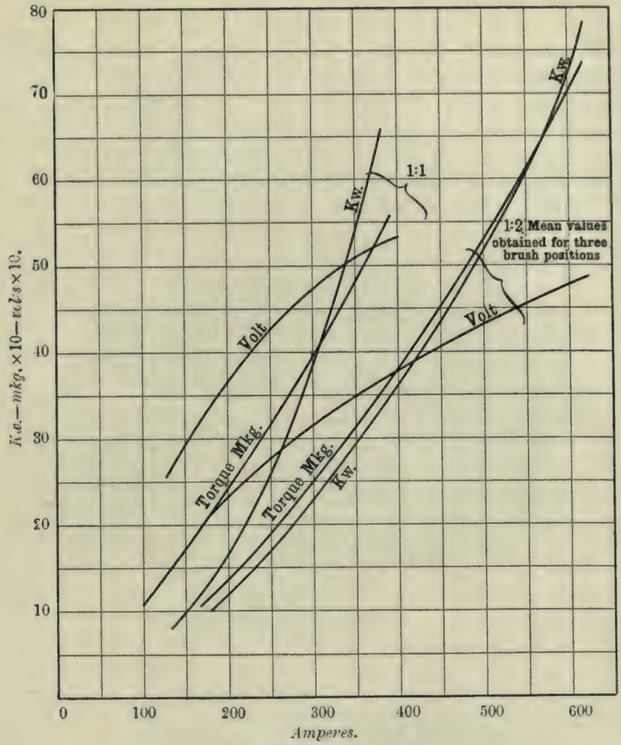


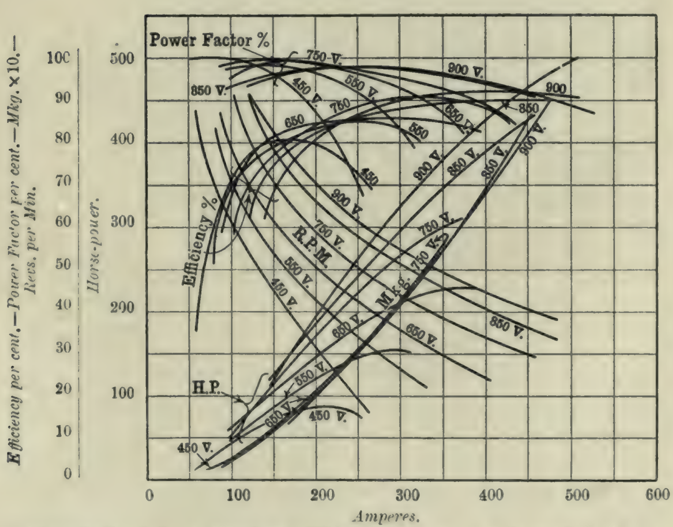
Fig. 171.—WE Motor of the A.E.G.

Figs. 170 and 171 are external views of the six-polar motor, WE 80, for 850 volts, 25 periods, 350 h.p., 450 revs. per min.



Hour Rating: 350 B.H.P.

FIG. 172.



Performance Curves of Winter-Eichberg Motor, W.E. 80. Hour rating, 350 B.H.P. at 850 volts, 450 revs. per min. (900 revs. per min. max.). 25 cycles, 6 poles.
NOTE.—Friction of axle bearings and gear not included.

FIG. 173.

(highest speed 900 revs. per min.). Figs. 172 and 173 are the performance curves of this machine for starting and running conditions.

External views of a WE 42 for 80 H.P., 25 periods, 750 revs. per min. (1,300 max.) and 650 volts are given in Figs. 174 and

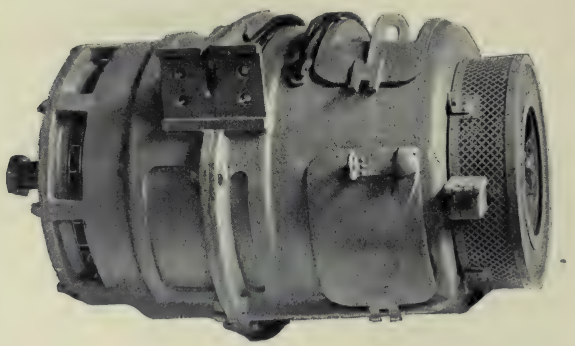


FIG. 174.

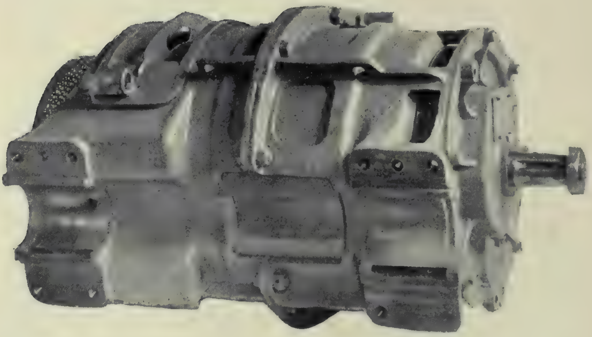


FIG. 175.

175. The number of poles is four. The pictures show the ventilating system employed with these machines.

The motors WE 51 V (Figs. 176 and 177) are those of the Hamburg City and Suburban Railways. The output is

115 H.P., 600 revs. per min., 750 volts, 25 cycles, number of poles = 4. In Fig. 180 commutating coils can be distinguished on two of the four poles (that in front and the upper pole). Figs. 178 to 180 show diagrams of the voltages of this motor.

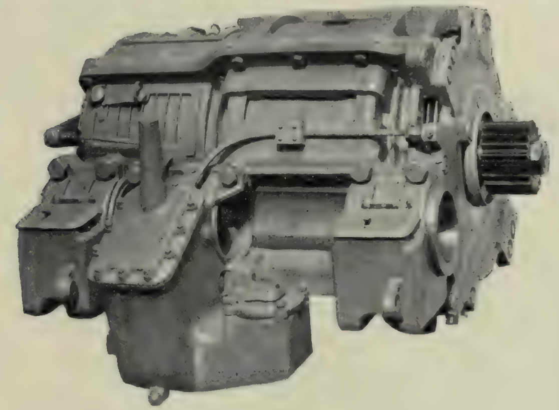


FIG. 176.

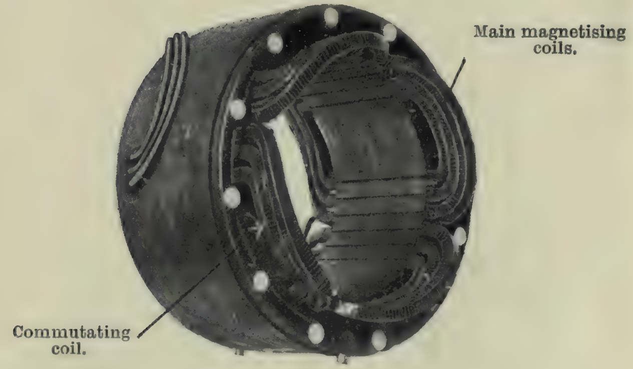
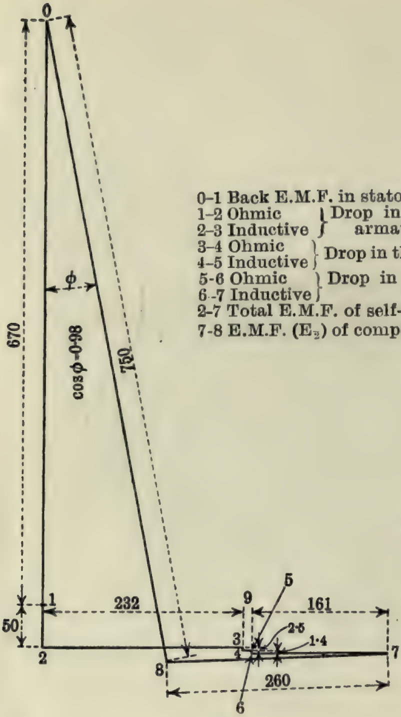


FIG. 177.

The two figures, 181 and 182, represent smaller motors, Fig. 181 a six-polar 40 H.P. machine for 40 periods and 600/1,000 revs. per min. Fig. 182 is a lift motor with six poles for 5 H.P., 800 revs. per min., 40 periods, with centrifugal switch.



0-1 Back E.M.F. in stator (E_a).
1-2 Ohmic } Drop in stator and short-circuited
2-3 Inductive } armature circuit (2-2) Fig. 120.
3-4 Ohmic } Drop in the transformer.
4-5 Inductive }
5-6 Ohmic } Drop in the exciter armature circuit
6-7 Inductive } (1-1) Fig. 120.
2-7 Total E.M.F. of self-induction E^l.
7-8 E.M.F. (E_2) of compensation.

FIG. 178.—WINTER-EICHBERG MOTOR, 115 P.S., 25 Cycles, 900 R.P.M., 750 Volts, 4 Poles.

Ratio of transformation of exciting transformer −0·75 : 1.

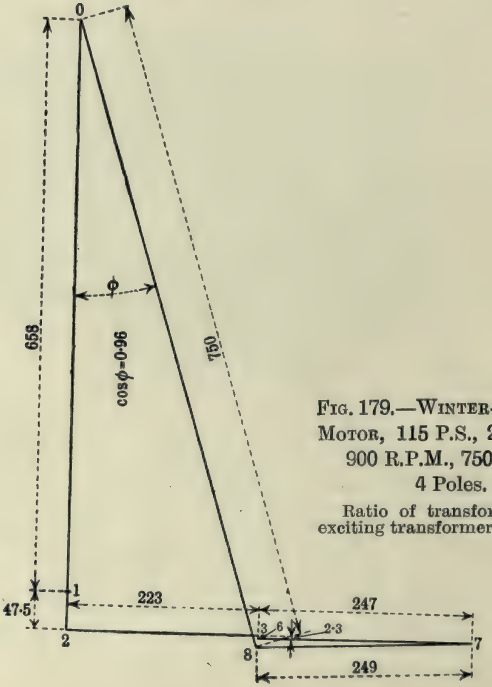


FIG. 179.—WINTER-EICHBERG MOTOR, 115 P.S., 25 Cycles, 900 R.P.M., 750 Volts, 4 Poles.

Ratio of transformation of exciting transformer 1 : 1.

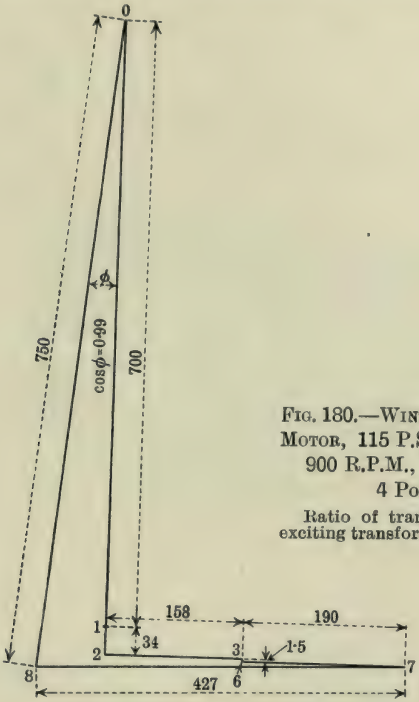


FIG. 180.—WINTER-EICHBERG MOTOR, 115 P.S., 25 Cycles, 900 R.P.M., 750 Volts, 4 Poles.

Ratio of transformation of exciting transformer 1:1.

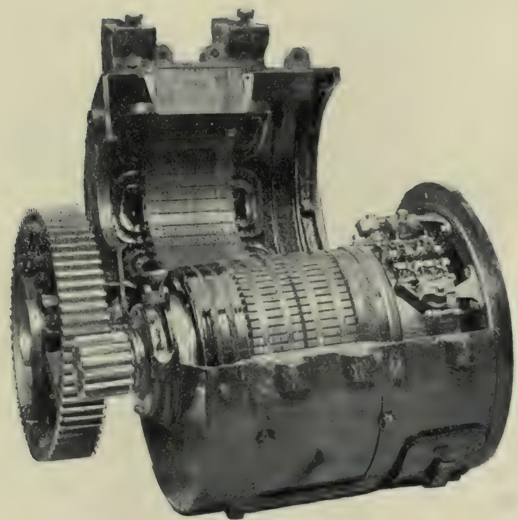


FIG. 181.

The *Fynn* motor, manufactured by Messrs. Alioth in Bâle, is shown in Fig. 183. This class of motor is standardised especially for lifts. As a rule they are supplied for 50 periods as a maximum, larger machines (over 10-15 H.P.) being built for 25 cycles as the highest frequency. The running and starting performances of an IMF 2 type for 1·25 H.P., 250 volts, 50 periods, 1,000 revs. per min., is given in Figs. 184 and 185. The motor has six poles.

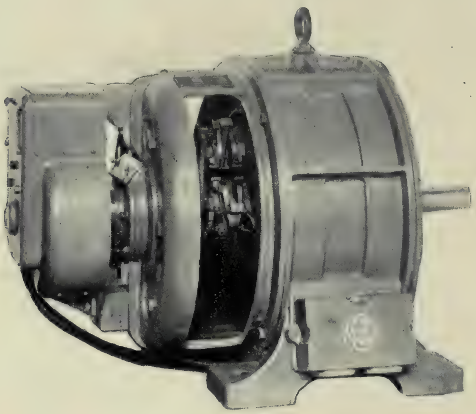


FIG. 182.

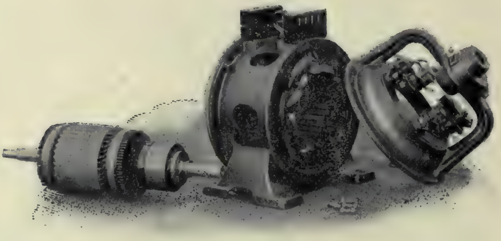


FIG. 183.

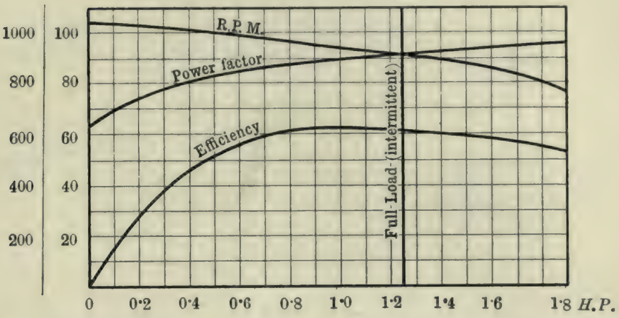


FIG. 184.—PERFORMANCE CURVES OF FYNN-ALIOTH SINGLE-PHASE MOTOR JMF$_2$.

6 poles, 1·25 B.H.P., 250 volts, 6·6 amperes, 50 cycles, 1,000 R.P.M.

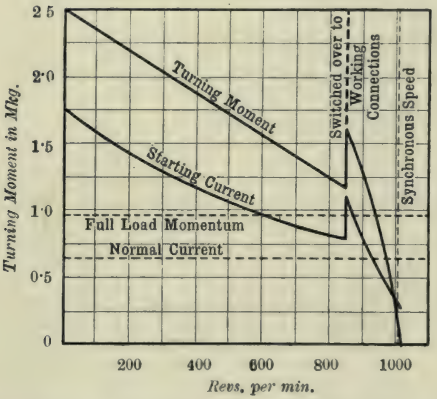


FIG. 185.—STARTING TORQUE OF FYNN-ALIOTH SINGLE-PHASE MOTOR JMF$_2$.

6 poles, 1·25 B.H.P., 250 volts, 6·6 amperes, 50 cycles, 1,000 R.P.M

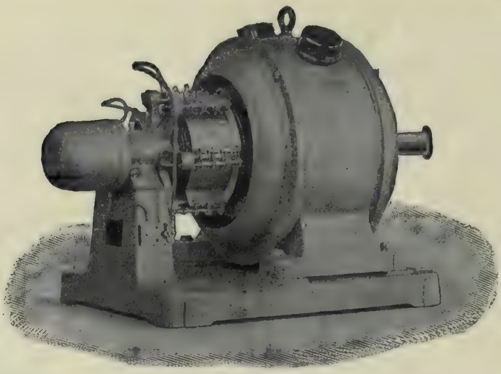


Fig. 186.

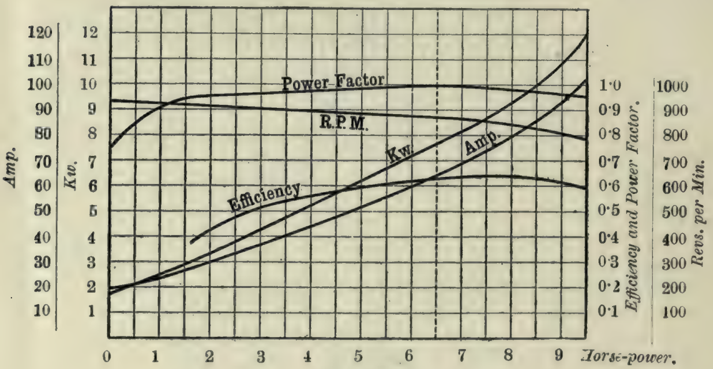


Fig. 187.

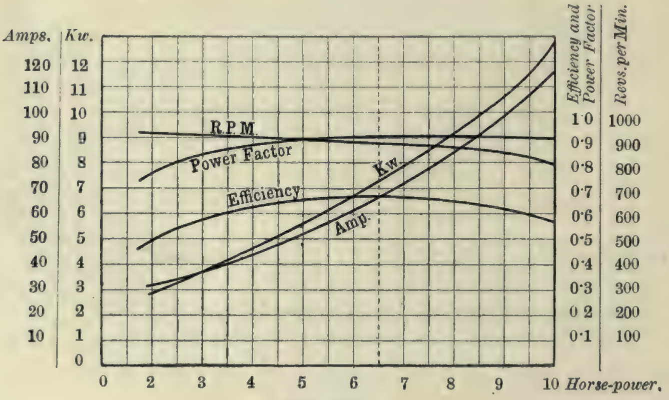


Fig. 188.

A view of the *Osnos* motor, the "double-circuit motor" as built by Messrs. *Lahmeyer* in Frankfort, is shown in Fig. 186. The performance curves Figs. 187 and 188 refer both to the same motor, type DGVI for 6½ H.P., 120 volts, 45 periods, the exciter voltage being 6 and 5 volts respectively. Figs. 189 and 190 are running and starting performances of a DG IX for 10 H.P., 220 volts, 50 periods.

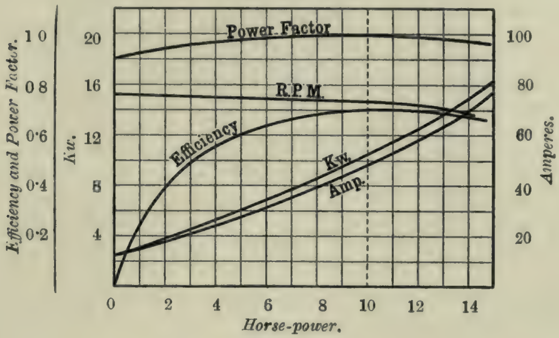


FIG. 189.

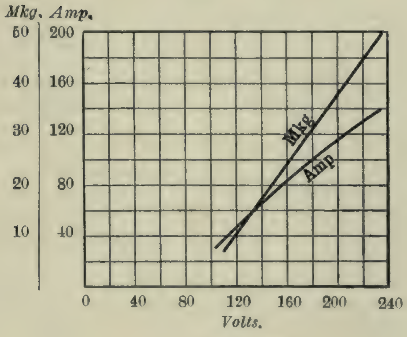


FIG. 190.

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THE
LEAKAGE OF INDUCTION MOTORS.

PREFACE.

Previous to the appearance of the series of Articles on Commutator Motors which form the basis of the preceding Book, the Author published in "The Electrician" the result of his experience on the Leakage of Induction Motors. This treatise contained many tables and curves intended for the handy use of the designer of Induction Motors, and from several quarters interest in the matter was expressed. Publication in book form was, therefore considered advisable, and—the matter being too short for a separate volume—it has been decided to attach it to the book on Commutator Motors.

R. GOLDSCHMIDT.

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THE LEAKAGE OF INDUCTION MOTORS.

An important problem of electricity which is so complicated that an exact solution is practically impossible, usually causes numerous approximate solutions to be put forward. This has been so with the predetermination of the leakage of induction motors, where exact calculation is almost excluded. The importance of the leakage problem is borne out by the fact that the overload capacity, the power factor and the starting torque—in fact, the whole performance of the motor —depend on it. All dimensions of the machine are partly or entirely influenced by it, so that the creation of approximate methods was a necessity.

Of the methods published, only those more recent ones need be considered which more or less take account of the leakage of the coil ends. We will mention

1. *Behn-Eschenburg*, who gives a formula consisting of three parts, the three coefficients of which have been found entirely empirically.

2. *Behrend's formula*, which originally possessed only a single empirical coefficient, but which has been enlarged by *Hobart*, and in his form contains three factors found by experiment.

3. *Adams' method*, which is based partly on calculation and partly on test.

The methods 1 and 2 are excellent for quick and approximate calculations. They are, however, not generally applicable, as only a few dimensions are taken into account. One is not safe from surprises in one or the other special case.

We shall go into the formula of Behrend and Hobart, which is so tempting by its simplicity, and we shall proceed to find the conditions on which it is based.

The method of Adams is rather complicated. It is, no doubt, correct to carry the calculation as far as Adams has done it, or rather, further still, before resorting to experiment, but the results might perhaps be more compact, as long formulæ containing the smallest dimensions are very difficult to handle, and do not make the calculation clear.

It is only recently that the importance of the coil end leakage has been recognised, the latter being often greater than that of the slot leakage. However, up to now no practical data have been published showing how the coil end leakage depends on the class of winding and the type of the coils, and from what point of view the best results can be obtained. Furthermore, nothing has been published as to how the leakage conditions are arrived at with two-phase and single-phase machines.

This treatise has been prepared with the object of developing methods for the calculations of the no-load current and the leakage of three-phase, two-phase and single-phase motors, taking account of all influences as far as possible. The preservation of simplicity in our methods will be a leading principle.

Table of Symbols.

a = Minimum width of teeth in centimetres.

b = Width of slots in centimetres.

c, C, k, K = Constants.

d = Diameter of rotor in centimetres.

e = Voltage.

f = Index for "iron."

g = Opening of slots in centimetres.

G = Lengths of the straight portion of the teeth.
h = Height of slots.
i = Current.
k, K = Constants and index for "short-circuit."
l = Length of iron in axial direction (core length).
m = Total number of slots.
n = Number of slots per phase per pole.
N = Number of lines of force.
p = Number of poles.
r = Radius.
R = Magnetic resistance, reluctance.
S = Total number of conductors per phase.
s = Number of conductors per slot.
t = Slot pitch.
T = Permeability for lines of force.
z = Number of slots per phase.
0 = Index for "no load."
1 = Index for stator.
2 = Index for rotor.
λ = Air-gap.
ν = Frequency per second.

I.—The Circle Diagram.

If the electrical and magnetic conditions of electrical machines—as, for instance, induction motors—are of a complicated nature when working under normal full load, the investigation is usually split up into two portions : (1) No load, (2) Short-circuit.

The " normal load " is " between both," and its conditions can be calculated from short-circuit and no load, or can be found by the construction of a diagram. A source of mistakes with these diagrams is the supposition that the occurrences at short-circuit and at no load can take place at the same time without disturbing one another.

For practical application and for comparison it is advantageous to replace the diagram and the formula by explicit tables giving figures for all practical cases and enabling quick and ready application.

We desire to represent by tables two relations:

1. The relation between power factor and coefficient of leakage at different loads. (Tables I. and II.)

2. The overload capacity dependent on the energy consumed by the motor at short-circuit. (Table III.)

We put i_0 = no-load current.
i_K = short-circuit current.
$i'_K = i_K / \sin \phi_K$.
$\cos \phi_K$ = power factor at short-circuit.

The ratio $i_0 / i'_K = \tau$ is very nearly equal to Heyland's leakage coefficient. It fixes the main dimensions of the diagram.

Table I.—*Power Factors of Induction Motors with Slip-rings.*

Leakage factor τ'.	Degree of load.								
	¼	½	¾	1	1¼	1½	1¾	2	2¼
0·03	0·85	0·93	0·95	0·95	0·945	0·935	0·925	0·895	0·805
0·05	0·74	0·865	0·91	0·92	0·92	0·915	0·905	0·88	0·79
0·07	0·66	0·81	0·875	0·90	0·905	0·90	0·89	0·87	0·78
0·10	0·565	0·73	0·825	0·86	0·87	0·87	0·865	0·845	0·77
0·13	0·505	0·66	0·76	0·81	0·83	0·84	0·84	0·825	0·75
0·16	0·46	0·615	0·715	0·77	0·80	0·81	0·81	0·80	0·735
0·20	0·42	0·55	0·66	0·725	0·76	0·78	0·785	0·78	0·72
0·25	0·385	0·51	0·615	0·68	0·72	0·745	0·755	0·755	0·70

Table II.—*Power Factors of Induction Motors with High-resistance Short-circuit Rotors.*

Leakage factor τ'.	Degree of load.								
	¼	½	¾	1	1¼	1½	1¾	2	2¼
0·03	0·84	0·935	0·95	0·955	0·955	0·95	0·94	0·925	0·86
0·05	0·715	0·86	0·91	0·93	0·93	0·93	0·92	0·905	0·845
0·07	0·635	0·80	0·875	0·90	0·91	0·91	0·905	0·895	0·835
0·10	0·525	0·71	0·805	0·85	0·87	0·88	0·88	0·87	0·815
0·13	0·47	0·64	0·75	0·805	0·835	0·85	0·855	0·85	0·80
0·16	0·425	0·59	0·70	0·765	0·80	0·82	0·83	0·83	0·785
0·20	0·41	0·55	0·655	0·725	0·76	0·79	0·80	0·805	0·77
0·25	0·375	0·505	0·60	0·675	0·72	0·75	0·77	0·775	0·75

Table III.—*Ratio between Maximum Output and Short-circuit Energy.*

$\cos \phi$	A	$\cos \phi_K$.	A	$\cos \phi_K$.	A
0·10	0·48	0·40	0·36	0·70	0·295
0·20	0·42	0·50	0·335	0·80	0·285
0·30	0·385	0·60	0·315	...	...

A certain magnetising component (about $i/2$) ought to be subtracted from the short-circuit current observed,* this component being required to produce the flux for interlinking primary and secondary windings at short-circuit. Our i'_K does not contain this component.

Dealing with test results it is easiest to introduce i_K instead of i'_K and to calculate with $\tau' = i_0/i_K$. Cos ϕ_K is of small influence on the power factors, especially so on those at normal load and small overloads.

Allowing, at 100 per cent. overload, an error of about 2 per cent., and at normal and smaller loads an error of about ½ per cent., we can give the relations between cos ϕ and τ' in the form of two tables. The two groups of machines represented by these two tables are

(*a*) Machines with slip-ring rotors. (Table I.)

$$\text{Cos}\ \phi_K = 0{\cdot}20 \text{ to } 0{\cdot}40.$$

(*b*) Machines with short-circuit high-resistance rotors. (Table II.)

$$\text{Cos}\ \phi_K = 0{\cdot}45 \text{ to } 0{\cdot}65.$$

These tables have been worked out for machines with 125 per cent. overload capacity. They naturally can be made use of with machines having other overload capacities. For instance, a machine with a maximum load of 4½ times normal the power factor at *normal* load is the same as that at half-load with a machine having a maximum load of 2¼ times normal. The scale of loads only has to be changed in the ratio of the given over-load capacity to 2¼. The value 2¼ has been chosen with a view to normal conditions with polyphase motors.

As the vector of the short-circuit current fixes the circle of the diagram, and the maximum output appears as a line between the circle and the chord (Fig. 1), the maximum output of the machine is proportional to the energy consumption in kilovolt-amperes at short-circuit, provided we multiply by a factor, A, only dependent on cos ϕ_K.

---

* See the Author's article dealing with this subject in the *Elektrotechnische Zeitschrift* for 1900.

If e is the voltage per phase, u the number of phases, we can put:—

$$\text{Maximum horse-power} = A \times u \times e \times i_K/746. \quad . \quad (1)$$

A is to be taken from Table III. A averages, for motors with slip rings, 0·39; for motors with short-circuit rotors, 0·32—*i.e., the maximum output of a motor is 32 to 39 per cent. of the energy in kilovolt-amperes it consumes under full voltage when the rotor is locked.*

To obtain an overload capacity of 125 per cent. the machine must be built so that the apparent short-circuit energy is equal to 6 to 7 times the normal output, or if the power factor at normal load is 0·87 and the normal efficiency, say, 0·84, the short-circuit current must be 4·3 to 5·2 times the normal current.

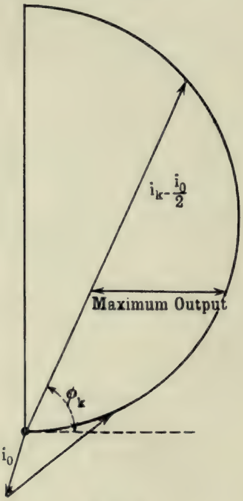

FIG. 1.

These figures can easily be corrected. They must be changed proportionately for different efficiencies, overload capacities, &c.

II.—The No-Load Current.

The no-load current of an induction motor consists of the magnetising component and the watt component for covering the losses at no-load. The magnetising component is by far the larger (Fig. 2). The power factor—*i.e.*, the ratio of the watt component to the total current—is rarely larger than 0·25. The magnetising current is, therefore, $\sqrt{1-0{\cdot}25^2}=0{\cdot}97$ of the no-load current. Practically both are equal.

That we are justified in neglecting the small difference is shown by the following consideration: A machine with a rotor diameter of 30 cm. has an air-gap of about 1 mm. If the radius of the rotor is only one-twentieth of a millimetre too small or too large, the air-gap becomes inexact within 5 per cent., and the error in the no-load current is almost equally large. With practical machines of the same type the no-load currents vary about 15 per cent. from one to the other.

We shall neglect the influence of the watt component on the no-load current altogether.

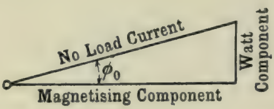

Fig. 2.

If a given winding is under the influence of a certain alternating voltage and frequency, a number of lines of force are produced, the maximum of which can be found from the equation

$$e=2{\cdot}22\times S\times\nu\times N\times 10^{-8} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

e = Root-mean-square value of the voltage.
N = Maximum number lines of force.
S = Number of conductors.
ν = Frequency per second.

With an induction motor we have to put:

e = Voltage per phase.
S = Total number of wires of all coils which are connected in series in each phase.
N = The maximum magnetic flux per pole.

This formula holds good only in the case where the magnetic flux is fully enclosed by the windings—*i.e.*, only in the case of one slot per phase per pole (Fig. 3). If the winding is distributed over several slots per phase per pole—for instance, over four (Fig. 4)—we have to consider this case as if four different windings were there, each occupying only one slot per phase and pole.

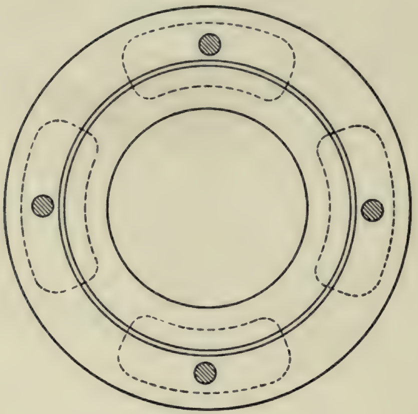

Fig. 3.

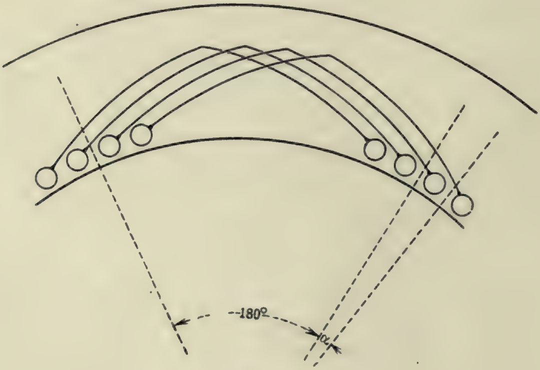

Fig. 4.

With all induction motors on no-load, practically a sinoidal revolving field is produced. This is effected by the co-operation of the different phases of the stator or indirectly through the influence of the rotor. The E.M.F.s produced in the four windings by the revolving field are, therefore, sinoidal and displaced in phase by the angle α between the slots (pole-pitch = 180 deg.) (Fig. 4).

If e is the voltage in each of the four windings and $S' = \frac{S}{4}$ the number of wires per winding, we find

$$e' = 2{\cdot}22 \times S' \times \nu \times N \times 10^{-8}.$$

The four e' must be added graphically (Fig. 5).

Carrying out this addition for different angles, α—*i.e.*, for different numbers of slots—we can put

$$e = 2{\cdot}22 \times k_0 \times S \times \nu \times N \times 10^{-8} \quad . \quad . \quad . \quad (2\text{A})$$

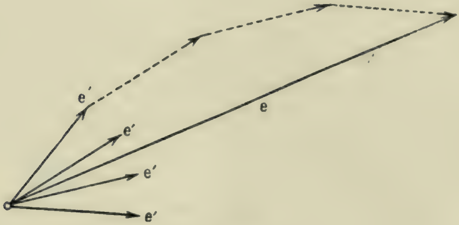

FIG. 5.

With one slot per phase per pole the winding constant k_0 is equal to unity. For two and more slots per phase and pole we may introduce an average figure for k_0 and put

$$\left.\begin{array}{lll} \text{For three-phase motors} & & k_0 = 0{\cdot}96 \\ \text{,, two-phase} & \text{,,} & k_0 = 0{\cdot}91 \\ \text{,, one-phase} & \text{,,} & k_0 = 0{\cdot}84 \end{array}\right\} \quad . \quad . \quad (2\text{B})$$

With single-phase current we assume that the winding is distributed over two-thirds of the inner surface of the stator.

Introducing these figures, we find

$$\left.\begin{array}{lll} \text{For three-phase motors} & & e = 2{\cdot}13 \times S \times \nu \times N \times 10^{-8} \\ \text{,, two-phase} & \text{,,} & e = 2{\cdot}02 \times S \times \nu \times N \times 10^{-8} \\ \text{,, one-phase} & \text{,,} & e = 1{\cdot}86 \times S \times \nu \times N \times 10^{-8} \end{array}\right\} \quad . \quad (2\text{C})$$

A special correction is necessary if the span of the coils is smaller or larger than the pole-pitch. This is a usual arrangement with open slots and machine-wound coils. Fig. 6 shows the coils of one phase with three slots per phase per pole and a reduction of span by one slot. The correction is easily calculated if we consider that the two sides of the coil which are embedded in the iron have E.M.F.s induced in them which have no longer the same phase, but are displaced from each other by an angle β.

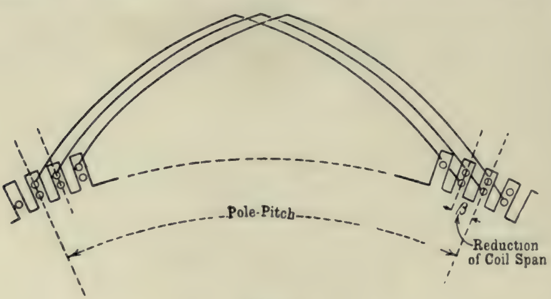

FIG. 6.

The tension per coil is (Fig. 7)

$$e = e' \times \cos\frac{\beta}{2}.$$

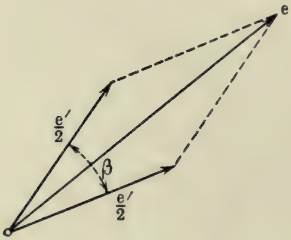

FIG. 7.

Formulæ (2C) are modified, a factor k_1 being introduced at the right-hand side, where $k_1 = \cos\frac{\beta}{2}$.

The reduction of span can be chosen very large up to about 90 deg. and less if it is desirable to save space for the coil ends.

A very short span of the coils, however, causes an imperfect field. The theoretically best figure for the reduction of span β is about 60 deg.

We may put as a practical average $\beta = 45$ deg. and find $k_1 = \cos\ (22\tfrac{1}{2}\ \text{deg.}) = 0{\cdot}92$, and our formulæ for reduced span become

$$\left.\begin{array}{lll}\text{For three-phase current} & e = 1{\cdot}95 \times S \times \nu \times N \times 10^{-8} \\ \text{,,}\quad \text{two-phase current} & e = 1{\cdot}85 \times S \times \nu \times N \times 10^{-8} \\ \text{,,}\quad \text{one-phase current} & e = 1{\cdot}71 \times S \times \nu \times N \times 10^{-8}\end{array}\right\} \quad . \quad . \quad (2\text{D})$$

We have to deal with a very short span in the case where the speed of the induction motor is to be altered by altering the number of poles. Figs. 8 and 9 show diagrammatically the connections for p poles and $p/2$ poles. The signs + and − indicate the direction of the current in the embedded portions of the coils. The change of the number of poles has been effected by reversing the current in the coils II. and IV., &c. The p-pole connection (Fig. 8) is quite normal, whilst with the $p/2$-pole connection (Fig. 9) the pole pitch (180 deg.) is double as large as before, the coil span being the same. Therefore, the reduction of the span is 90 deg. and the coefficient k_1 becomes cos (45 deg.) = 0·70.

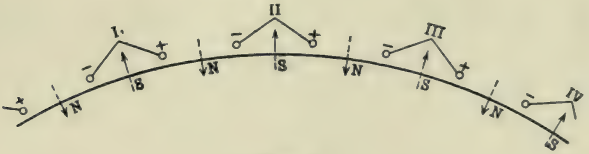

FIG. 8.

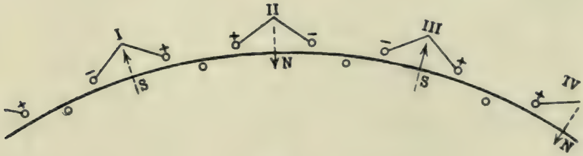

FIG. 9.

Having calculated the total number of lines per pole N, we find the mean density in the air and in the teeth by dividing

N by the air section and tooth section per pole respectively. This mean value is of small significance for the no-load current, the latter being influenced by the maximum density, which, with sinoidal distribution of the flux, is $\pi/2$ times the mean density.

We put:—

Q_t = Minimum of the total section of all teeth.
p = Number of poles.
$\frac{Q_t}{p}$ = Minimum of the tooth section per pole.
Q_i = Total air section.
B_l = Maximum density in the air.
B_t = Maximum density in the teeth.

Then the general formulæ for calculating these densities become

$$B_l = C_0 \times \frac{e \times p \times 10^8}{\nu \times S \times Q_l}. \qquad (3\text{A})$$

$$B_t = C_0 \times \frac{e \times p \times 10^8}{\nu \times S \times Q_t}. \qquad (3\text{B})$$

The constant becomes

For three-phase $C_0 = 0{\cdot}74$.
,, two-phase $C_0 = 0{\cdot}78$.
,, one-phase $C_0 = 0{\cdot}85$.

With reduced span of the coils these figures must be divided by k_1.

The ampere-turns required to drive the magnetic lines of density B_t twice through the air-gap λ are

$$AW_l = \frac{10}{4 \times \pi} \times B_l \times 2 \times \lambda = 1{\cdot}6 \times B_l \times \lambda. \qquad (4)$$

The path of the magnetic lines in the iron consists of the path in the yoke of stator and rotor and that in the teeth.

The ampere-turns AW_y for the yokes can be calculated in the well-known way by means of a saturation curve for the iron after finding the mean density $B_y = N/2Q_y$. Q_y = yoke section. The mean length of the magnetic path for the yoke can be put equal to the arc over one pole. AW_y is usually only considerable with machines having few poles and low frequency.

The ampere-turns for the teeth, AW_t, evidently depend not only on the maximum density (point *a* in Fig. 10), but also on the density in the rest of the teeth. Constructing a curve for the density in a tooth, and starting from the section *a* with the density B_t, we find that the density drops hyperbolically because the tooth section increases proportionally to the radius **r** (Fig. 10) and "Section × density" is constant.

We put r_t=AB, n_1=number of slots in the stator, G=the length of the straight portion of the tooth, $B_x/B_t r_t = (r_t + x)$; or after transforming we find for x = one centimetre

$$B_{1\text{cm.}}/B_t = \frac{1}{1 + 2\pi/an_1}.$$

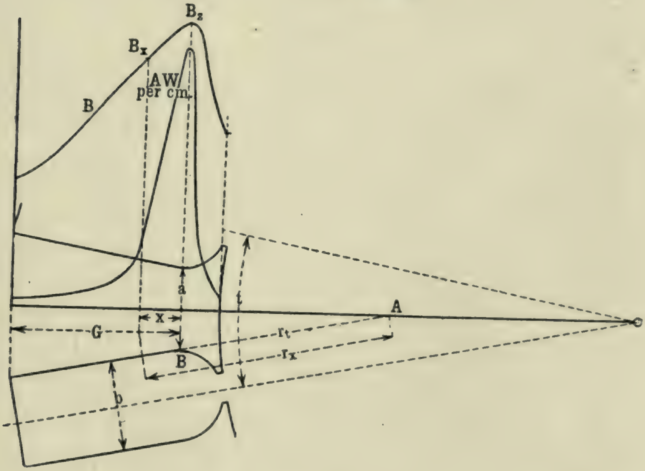

Fig. 10.

This expression is the decrease of density per one centimetre length of tooth, and it only depends on the sum of all "minimum thicknesses" of teeth, $n_1 \times a$—*i.e.*, with given $n_1 \times a$ the form of the B curve is fixed.

Besides the B curve, Fig. 10 shows also the curve of the ampere-turns per centimetre belonging to the different densities. The integral of these AW per centimetre for the amount G is the total number of ampere-turns required for this part.

In consequence of the very quick dropping off of the AW curve in Fig. 10 (*for machines with* $n_1 \times a < 70$ *centimetres*) *the integral value of the tooth ampere-turns is within practical limits only very little dependent on G.*

We could draw a set of saturation curves for different values of $n_1 \times a$ which make the ampere-turns per tooth directly dependent on B_t. One fact, however, makes all these calculations inexact. As soon as the tooth on the place of the maximum density begins to get saturated the neighbouring teeth take up a larger number of lines of force, so that the field distribution ceases to be sinoidal and the maximum density will be smaller than calculated. Another source of error is due to the lines of force entering the slots themselves when the teeth begin to get saturated.

If the density in the teeth in stator and rotor is approximately the same, Table IV. can be used with advantage. This table gives average figures for the ampere-turns required to magnetise two stator teeth plus two rotor teeth made dependent on B_t only, and without regard to the length of the teeth and the radius of the rotor. These values are average figures from a large number of curves observed on actual machines. They cannot claim great accuracy, but in most cases it is sufficient. At any rate, they are very convenient for calculation.

Table IV.

B_t Max. tooth-density. (Formula 3)	Total amp.-turns for teeth.	B_t max. tooth-density. (Formula 3)	Total amp.-turns for teeth.	B_t max. tooth-density. (Formula 3)	Total amp.-turns for teeth.
15,000	50	18,000	180	21,000	340
16,000	90	19,000	230	22,000	390
17,000	125	20,000	290	23,000	450

The ampere-turns which we have calculated are required to drive the magnetic lines at the places of the maximum density twice through air-gap and teeth, as well as through the yokes. The distance between two points of maximum density, *a* and *b*, is one pole-pitch. Therefore, the whole of the ampere-turns per pole are utilised for the production of the magnetic lines (Fig. 11).

With *three-phase motors the magnetising current* in one phase has its maximum $i_{0\,\text{max.}}$, while in the two others the current is $\frac{1}{2} \times i_{0\,\text{max.}}$ The maximum of the density is, therefore, in the middle of the coil with the large current, so that for this distribution of current—viz., every 60 deg.,

$$\text{AW} = 2 \times i_{0\,\text{max.}} \times \frac{\text{S}}{p}.$$

If the field has moved 30 deg. forward

$$\text{AW} = 2 \times 0{\cdot}87 \times i_{0\,\text{max.}} \times \frac{\text{S}}{p}.$$

We put
$$\text{AW} = 1{\cdot}94 \times i_{0\,\text{max.}} \times \frac{\text{S}}{p},$$

or, introducing the R.M.S. value $i_0 = \frac{1}{\sqrt{2}} \times i_{0\,\text{max.}}$,

$$i_0 = 0{\cdot}37 \times \frac{\text{AW}}{\text{S}} \times p. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5\text{A})$$

With two-phase current we find

$$i_0 = 0{\cdot}65 \times \frac{\text{AW}}{\text{S}} \times p. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5\text{B})$$

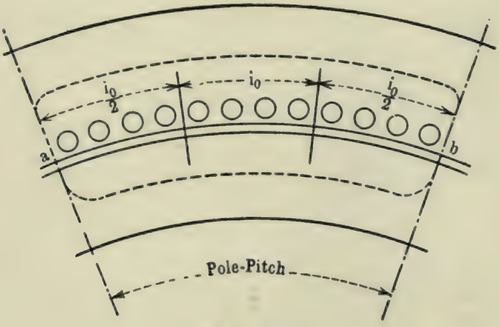

FIG. 11.

With *single-phase motors* the magnetising current is twice as large as is the case with two-phase machines. This relation is not only a theoretical one, but is amply confirmed by practical experience. Therefore

$$i_0 = 1{\cdot}30 \times \frac{\text{AW}}{\text{S}} \times p. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5\text{C})$$

If we divide the no-load current i_0 into two parts, that for the air-gap (i_{0l}) and that for iron (i_{0f}), we can calculate the former by means of a single formula, gained by combining the equations 3, 4 and 5. We find

$$i_{0l} = C'_0 \times \frac{e}{\nu} \times \frac{p^2}{S^2} \times \frac{\lambda}{Q_l} \times 10^8. \quad . \quad . \quad . \quad . \quad . \quad (6)$$

With three-phase current $C'_0 = 0{\cdot}43$.
With two-phase current $C'_0 = 0{\cdot}88$.
With single-phase current $C'_0 = 1{\cdot}76$.

When calculating the air section Q_l we have to take account of the openings in the slots. *If the slots are partly closed*, the opening being about 15 to 25 per cent. of the rotor circumference, *we subtract 85 per cent. of the sum of all openings in the stator* ($n \times g_1 \times 0{\cdot}85$) *from the circumference* (πd) and multiply this value by the coefficient $(1 - 0{\cdot}85 \times g_2/t_2)$:

$n_1 n_2$ = number of slots in primary and secondary.
$t_1 t_2$ = slot pitch.
$g_1 g_2$ = openings of slots primary and secondary.

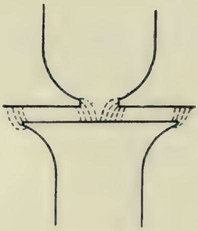

Fig. 12.

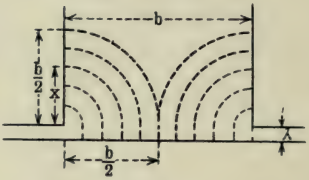

Fig. 13.

By introducing the factor 0·85, the fringe of the lines at the corners of the teeth shall be taken account of (Fig. 12). To obtain the air section we have to multiply by the actual length of the core l, inclusive of insulation between the discs.

With perfectly open slots in the stator, and partly closed ones in the rotor, we have to go into the calculation of the spreading of lines of force into the slot portion (Fig. 13). We utilise for this purpose diagrams of the magnetic lines constructed after those found hydraulically by Prof. Hele-Shaw. With the small air-gaps we have to deal with in induction motors we can form the magnetic lines of quarter circles with a radius proportionate to the distance from the corner x (Fig. 13), and of a straight portion equal to the air-gap λ (Fig. 13). The length of the path is, therefore, generally $\lambda + \frac{\pi}{2} \times x$; the permeance of a tube of lines entering the tooth in the distance x from its corner is, therefore,

$$T'_0 = \frac{dx \times l}{(\lambda + 1{\cdot}56 \times x)}.$$

If we integrate this value from 0 to $b/2$ and multiply by 2 we find the total permeance per slot

$$T'_0 = 1{\cdot}28 \times l \times lg\left(1 + 1{\cdot}56 \times \frac{b}{2\lambda}\right).$$

The magnetic reluctance of the path of fringe lines in the slot is equal to

$$R'_0 = \frac{1}{1{\cdot}28 \times l \times lg\left(1 + 1{\cdot}56 \times \frac{b}{2\lambda}\right)}.$$

If we subtract from R'_0 the amount x/bl—viz., the resistance of the path where the lines are practically parallel (Fig. 13)—we find that the remaining portion is a constant amount within practical limits. This constant amount is about 0·25, so that we can put

$$R'_0 = \frac{0{\cdot}25 + \frac{\lambda}{b}}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The value 0·25 agrees very well with figures gained independently direct from actual machines.

We carry out our calculation in such a way that we first deduct from the rotor circumference the sum of all slot openings, and then multiply this amount by $(1 - 0{\cdot}85.\ g_2/t_2)$ and l. This is the section for the magnetic lines under the teeth themselves. Dividing the air-gap λ by this section, we find the magnetic resistance of the air under the teeth. R''_0, R'_0 and R''_0 are in parallel, and, therefore, the total magnetic resistance—viz., the value $\frac{\lambda}{Q_l}$ in formula (6)—for the magnetising current for air i_{0l}, is

$$R_0 = \frac{\lambda}{Q_l} = \frac{1}{\frac{1}{R'_0} + \frac{1}{R''_0}}.$$

The influence of the openings in the rotor slots on R'_0 has been neglected, because it is very small.

III.—The Leakage.

If we deduct from the current of an induction motor under normal working load an amount equal to the magnetising current (no-load current), a current component is left in the stator which is not capable of producing a magnetic flux through the rotor. The rotor current itself prevents this, assuming such a value that the secondary ampere-turns are equal to the primary ones minus the no-load ampere-turns. The latter can never quite disappear, as they have to produce the magnetic flux interlinking rotor and stator. For the purpose of the calculation of leakage, however, we have to disregard them altogether. We replace them by connecting rotor and stator winding in series, both being given equal numbers of turns—*i.e.*, we send the energy into the rotor direct instead of transmitting it with the help of a transformation flux.

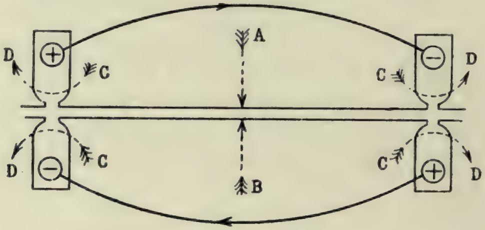

Fig. 14.

Fig. 14 shows a stator and a rotor coil being placed exactly opposite one another and connected in series in such a way that in the direction AB no magnetic flux can be produced. All magnetic lines created by the two coils have to take the following "leakage paths":—

(A) The path across the slots (CD, Fig. 14).
(B) The path across the slot opening, utilising the face of a tooth opposite this opening (FG, Fig. 15).
(C) The space round the coil ends (HK, Fig. 16).

We will consider these three classes of leakages separately, as path (A) only depends on the form of the slot, path (B) only on the relation between primary and secondary slots, and path (C) only on the form of the coil ends.

There is a fourth path for the leakage lines, which we shall refer to when dealing with leakage (B).

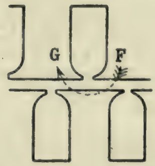

Fig. 15.

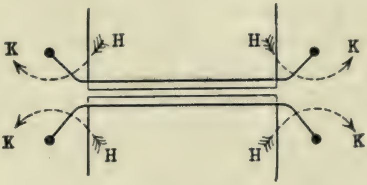

Fig. 16.

(A) *Slot Leakage.*

Fig. 17 shows four slots carrying current of one and the same phase; the other slots of the machine may be considered as empty. If s is the magnetic resistance for the lines on their way from *a* to *b* or from *b* to *c*, &c., and if, further, I is the number of ampere-turns per slot, the magnetic flux round every slot is $M = I/s$.

The resistance of the iron has been neglected. We have been following a path *a-b-f-g-a* or *b-c-h-f-b*, &c. The teeth *b*, *c*, *d* carry two magnetic fluxes, which are equal and opposite to one another—*i.e.*, the magnetic flux in them is really zero. M passes immediately from *a* to *e*, and the teeth *a* and *e* have to carry the total flux. Also, in the case of a completely wound induction motor, only two or four teeth conduct the leakage flux back into the yoke.

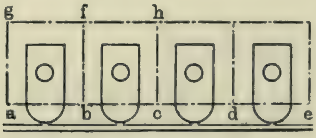

Fig. 17.

Following the path *a-e*, we could have put $M = \frac{4 \times I}{4 \times s}$.

It must be clearly understood that the condition for the phenomenon *that only the first and the last tooth conduct the leakage flux* is: That the resistance of the iron is practically zero. If the teeth *a* and *e* get saturated, the teeth *b* and *d* begin to conduct lines.

In spite of widely open slots in stator and rotor we obtain in such cases bent short-circuit curves, the bending being far less pronounced than we are used to with saturation curves.

This is due to the teeth *b* and *d* only gradually coming into use as the teeth *a* and *e* get saturated. With machines having only small leakage in the coil ends this phenomenon is particularly noticeable. Fig. 18 shows a bent short-circuit curve together with a magnetising curve taken with open rotor.

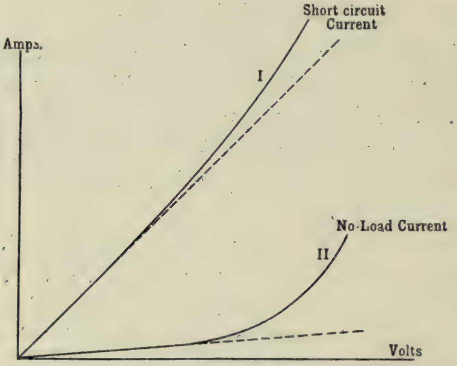

FIG. 18.

We notice that with a voltage where curve II. does not show any bend at all a distinct bend shows up in curve I.

Bent short-circuit curves can naturally also have their cause in other circumstances—for instance, in closed slots. With machines having much leakage and a high rotor resistance, the magnetising component at short-circuit is considerable, so that the saturation of the no-load current may show up in the short-circuit current. We will assume that at short-circuit the teeth are practically unsaturated. Then we may put the slot leakage directly proportional to the number of slots, entirely independent of the arrangement of the winding and any mutual induction from one slot to the other. If s is the number of wires per slot, and the number of slots per phase and pole—for instance, 4—the coefficient of self-induction is equal to a constant $\times 4 \times s^2$. If we should add a fifth slot of equal form and equal number of wires, the coefficient of self-induction is equal to a constant $\times 5 \times s^2$—*i.e.*, it is increased in the ratio of the slots 5/4, and not proportional to the square of the wires per phase and pole $(5/4)^2$.

We will now calculate the magnetic resistance, or rather its reciprocal value, the *permeance* per centimetre slot length. It may be noted that the permeance can be defined as the number of lines of force per $4 \times \pi/10 = 1{\cdot}25$ ampere-turns. We will consider, first, *partly-closed slots.*

Two classes may be distinguished, those with round and those with straight tooth horns (Figs. 19A and 19B). The form of the horns can, however, have no appreciable influence on the leakage; at any rate, we do not make a great mistake if we replace the round horn by a straight one, as indicated in Fig. 19A. Further, the angle ω is of only secondary influence in the calculation of the leakage. We will disregard altogether the form of the slot. Regardless of its being closed by a straight line 1—2 (Fig. 19B), or by a broken line, 7—8—9 (Fig. 19C), we shall base our calculations once for all on the form in Fig. 19B. The dimension m (Fig. 19C) is generally about 0·5 mm. to 1 mm. However, we must not load up our calculation with such small detail, and therefore we put $m = 0{\cdot}65$ mm.

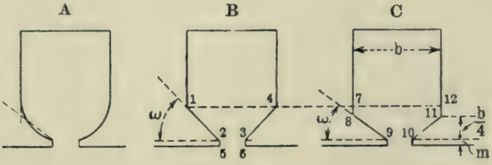

Fig. 19.

Calculation shows that the error caused through our simplification is very small indeed.

To fix the dimension of the slot we only need know the slot height h, the slot width b, and the slot opening g. Usually the winding fills a portion of the narrow part of the slots. The number of ampere-turns, however, which can be placed in this portion is too small to have a considerable influence on the distribution of lines. We therefore consider the narrow portion of the slot as free from winding.

Part 1 (Fig. 20), with parallel sides, is filled uniformly with winding (Fig. 21). A simple integration shows that the permeance of this portion per centimetre slot is

$$T'_{K1}=\frac{1}{3}\times\frac{h'}{b},\ h'=\text{height of the straight part}=h-\frac{b}{4}.$$

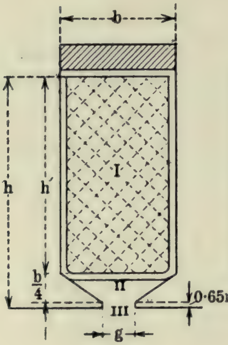

FIG. 20.

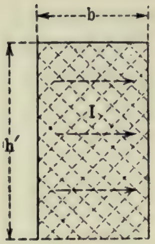

FIG. 21.

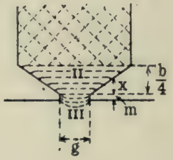

FIG. 22.

In the cone portion of the slot (Fig. 22) the magnetic lines can be considered as straight. Though they have the tendency to bend out circular (Fig. 23A), they are pressed back into a more or less straight form through the influence of the ampere-turns in part 1 (Fig. 23B).

The length of the magnetic line in the distance x from the opening g is

$$\left[g+(b-g)\times\frac{x}{0{\cdot}25\times b}\right],$$

and the permeance $T'_{K1}=\dfrac{0{\cdot}25}{1-\dfrac{g}{b}}\times\log(b/g)$.

For the third portion of the leakage in the slot opening (Fig. 24) we have to take account of the spreading of the lines right into the air-gap. We draw diagrams of the magnetic lines on a large scale for part 3 and calculate the permeance by splitting up the flux into tubes of lines of force. We find for

$g = 1$ mm., $T'_{K3} = 0{\cdot}83$,
$g = 2$ mm., $T'_{K3} = 0{\cdot}44$,
$g = 3$ mm., $T'_{K3} = 0{\cdot}30$,
$g = 4$ mm., $T'_{K3} = 0{\cdot}25$.

The formulæ for T'_{K1} and T'_{K2}, together with this table for T'_{K3}, enable us to calculate the slot leakage. The calculation, however, is still too complicated. Joining T'_{K2} and T'_{K3} together as "horn leakage," we can draw for the latter curves, making $T'_{K2} + T'_{K3}$ dependent on b for different values of g. If we carry this out for $g = 1$, 2, 3 and 4 mm., we find that within practical limits the difference in the permeances for the different values of g is constant and independent of the slot width b. Consequently we are able to base our calculation on a certain value for g which we consider as "normal," and to take account of other values of g by adding or subtracting a small correction. We choose *$g = 2$ mm. as normal value* and take account of the following corrections, which are *to be added* to the permeance calculated when g has other values than 2 mm.:

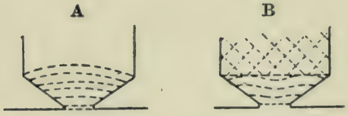

FIG. 23.

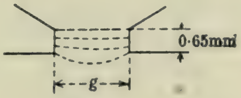

FIG. 24.

Table V.

g.	Correction.	g.	Correction.
1·0 mm.	$= +0{\cdot}53$	3·0 mm.	$= -0{\cdot}18$
1·5 mm.	$= +0{\cdot}22$	3·5 mm.	$= -0{\cdot}24$
2·0 mm.	$= 0$	4·0 mm.	$= -0{\cdot}28$
2·5 mm.	$= -0{\cdot}10$	4·5 mm.	$= -0{\cdot}30$

With totally closed slots the correction is from $+2{\cdot}5$ to $+3{\cdot}5$.

The curves in Fig. 25 give, ready for practical use, the permeance T'_K per centimetre slot length dependent on the slot width and for different slot heights.

If, for instance, the slot height is $h = 32$ mm.,
the slot width $b = 12$ mm.,
the slot opening $g = 3 \cdot 25$ mm.,

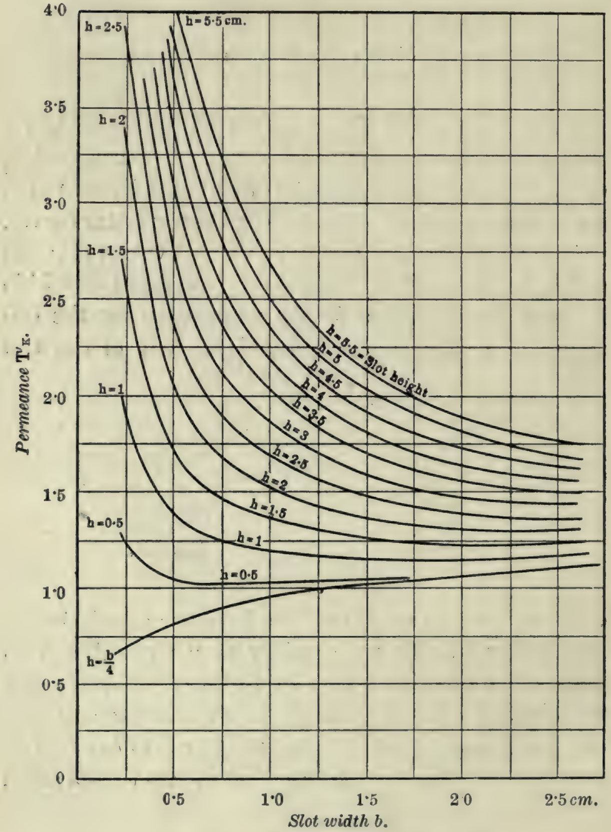

FIG. 25.—PARTLY CLOSED SLOTS.

Permeance per cm. slot length T'_K. Slot opening 2 mm.

we take from the curves the permeance per centimetre slot length $T'_K = 1 \cdot 75$. According to Table V., the correction for $g = 3 \cdot 25$ mm. is 0·20; therefore, $T'_K = 1 \cdot 75 - 0 \cdot 20 = 1 \cdot 55$.

It is easiest for calculation to join the leakage of the rotor slots, together with that of the stator slots, in such a way that, for determining the total permeance, only a multiplication by the primary number of slots n_1 and the core length is necessary. If $n_1 = n_2$ we can simply add the secondary permeance to the primary one. In a case where n_1 and n_2 are not equal, the secondary T'_K is to be multiplied by the ratio of slots n_1/n_2 before adding it to the primary T'_K.

In the above example we found T'_K for the stator alone equal to 1·55. T'_K for the rotor alone may be 1·30. If, further, $n_1 = 72$ and $n_2 = 48$, we find the permeance of the motor due to the slot leakage per centimetre slot length,

$$T'_K = 1{\cdot}55 + 1{\cdot}30 \times \frac{72}{48} = 3{\cdot}50.$$

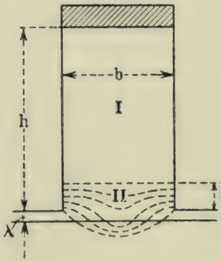

Fig. 26.

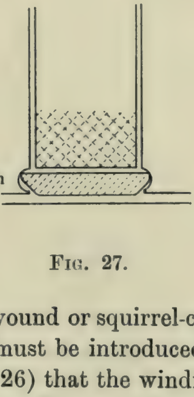

Fig. 27.

With single-phase motors with wound or squirrel-cage rotors only two-thirds of the rotor slots must be introduced as n_2.

With open slots we assume (Fig. 26) that the winding begins 0·5 cm. from the slot corner. We shall treat this part of the slot (part 2) separately from the slot filled with winding (part 1). For part 1 we can put again

$$T'_{K1} = \frac{h - 0{\cdot}50}{3 \times b}.$$

In part 2 the lines of force either pass directly from one side of the slot to the other, or they use the armature iron as a bridge, whichever way offers the smallest magnetic resistance. The simplest way, perhaps, to obtain values for the permeance is the construction of pictures of magnetic lines and the splitting up into magnetic tubes. Bearing in mind that the magnetic lines have the tendency to take the shortest path and to obtain as large a section of passage as possible, errors in the design of the magnetic tubes will be very small.

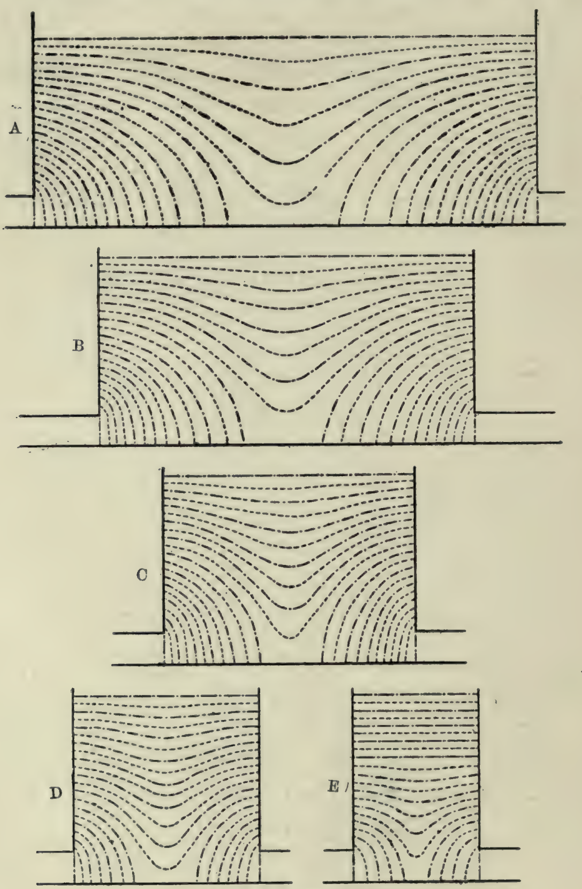

Fig. 28.

If at one place the section of a tube has been assumed too large, this error is balanced through the reduction of the sections of the neighbouring tubes. I have, at times, calculated the permeance of "space 2" for several distributions of the field which differed considerably one from the other, and I found only very small differences in the total permeance.

As an example I have reproduced in Figs. 28A, B, C, D, E, the diagrams of lines of force for 16, 12, 8, 6 and 4 mm. slot widths and 1 mm. air-gap (λ). Similar diagrams have been designed for different values of λ. The air-gap of 1 mm. is to be considered as "normal value." The curve in Fig. 29 for $h=0{\cdot}50$ cm.

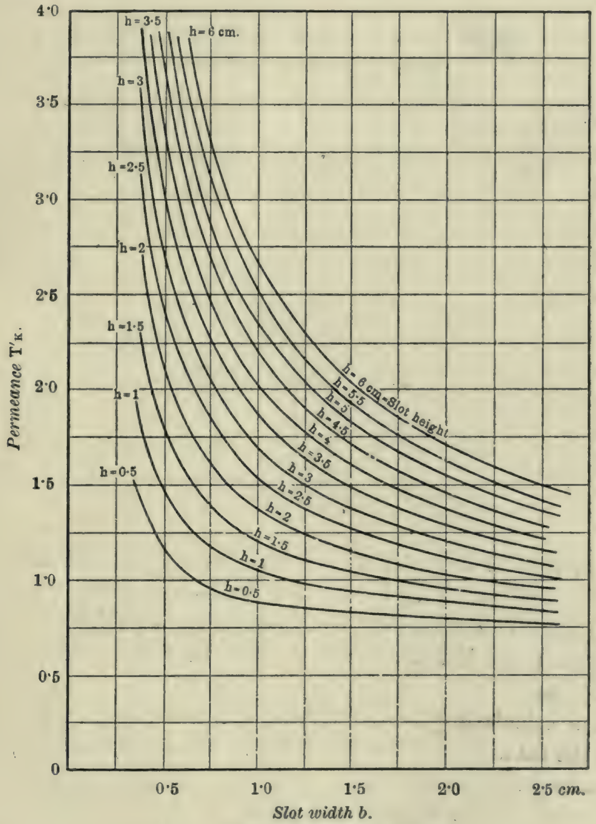

FIG. 29.—OPEN SLOTS.

Permeance per cm. slot length T'_K. Air-gap 1 mm.

gives the permeance of leakage T'_{K2} per 1 cm. length of slot and an air-gap of 1 mm. as derived from these diagrams.

The correction for other air-gaps is a very simple one. I found that 100 per cent. alteration in λ alters the permeance about 0·10 in accordance with Table VI.

Table VI.

Air-gap.	Correction.
0·50 mm.	+0·10
1·0 mm.	0
1·5 mm.	−0·05
2·0 mm.	−0·10

Joining together again T'_{K1} and T'_{K2} to form the total value of the slot leakage per centimetre slot length T'_K, we find *the curves Fig. 29, which are ready for practical use.* λ is equal to 1 mm. in these curves.

As an example we will assume a slot height, $h = 32$ mm.; slot width, $b = 12$ mm.; air-gap, $\lambda = 1·5$ mm.

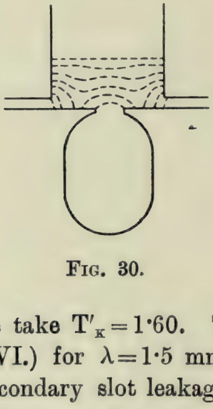

Fig. 30.

From Fig. 29 we take $T'_K = 1·60$. Taking account of the correction (Table VI.) for $\lambda = 1·5$ mm.—viz., 0·05, we find $T'_K = 1·55$. The secondary slot leakage is to be joined with the primary one in the same way as we have done for partly closed slots.

When calculating the leakage of the stator slots we have taken the rotor iron as free from slots. We ought, however, to have considered that the path of the leakage lines going from one side of the slot through the rotor to the other side is interrupted as soon as a rotor slot is opposite a stator slot (Fig. 30). Consequently, the permeance would be reduced. On the other hand, a leakage flux is formed round the opening of the rotor slot which tends to equalise this reduction, so that the alterations in the permeance during rotation are in reality very small, and our curves give values which check very well with actual tests.

If the conductors do not consist of thin wires, but of solid bars, the self-induction of the parts which are close to the slot opening (B) is larger than the others near (A). The calculation of an equivalent value of the leakage is simple. I have found, however, that in most practical cases it is not worth while to take account of the unequal distribution of the self-induction. The influence of the uneven current distribution in the bars on the self-induction itself is only small, though, as has been shown by Field, the copper losses with very narrow and deep slots can increase very considerably through this lack of uniformity.

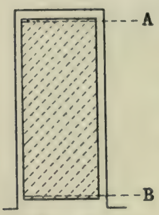

FIG. 31.

(B) *The Zigzag Leakage.*

For the second kind of leakage the name "zigzag leakage" has become usual in practice as well as in literature.

To start with, we will assume that the number of slots is the same in the primary and secondary, and that the openings of the slots are very small indeed.

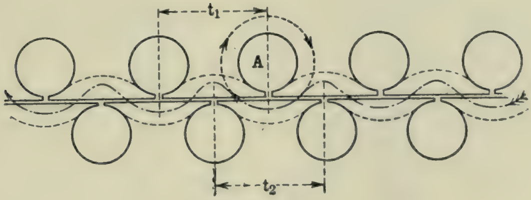

FIG. 32.

If stator and rotor slots have relative positions, as shown in Fig. 32, a stream of magnetic lines flows round the slots in zigzag form. For their calculation we assume, as before, magnetic circuits closed round the slots, either one half round the primary slot and the other round the secondary, or only round the primary, as indicated in Fig. 32 with slot A. Both modes of calculation lead to the same result. We therefore choose the simplest by assuming that the zigzag leakage lines close round the primary slots only.

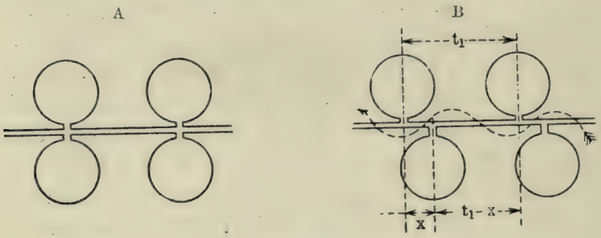

Fig. 33.

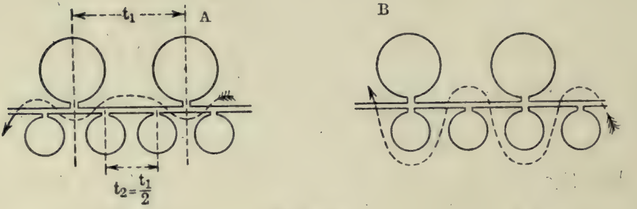

Fig. 34.

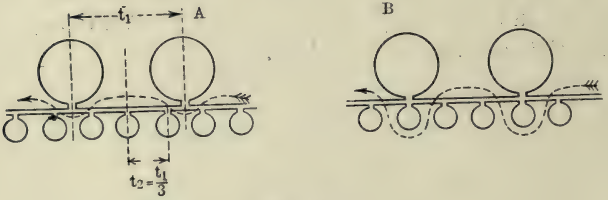

Fig. 35.

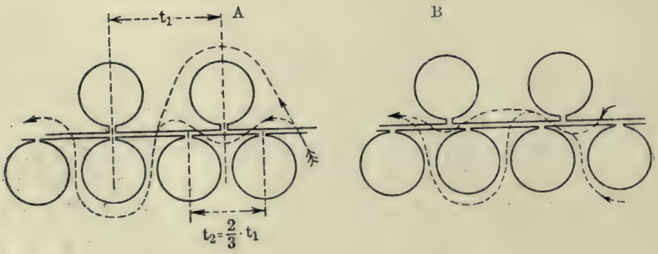

Fig. 36.

In the position Fig. 32 the permeance is largest—viz., $T''_{\text{K max.}} = \frac{t_1/2}{2\lambda} = 0\cdot25 \times t/\lambda$ per slot and per centimetre core length. In Fig. 33A the zigzag permeance is zero. In an intermediate position (Fig. 33B) $T''_{\text{K}x} = x/t_1 \times (t_1 - x)/\lambda$. The average value is

$$T''_{\text{K}} = \frac{2}{t} \times \frac{1}{\lambda \times t} \int_0^{\frac{t}{2}} (x \times t_1 - x^2) \times dx = 0\cdot167 \times t_1/\lambda.$$

If the number of slots in the rotor is double as large as that of the stator the former contains in each slot only half as many ampere-turns as the latter, as the total number of ampere-turns are equal. In the rotor position Fig. 34A the permeance is a maximum, viz., $0\cdot125 \times t_1/\lambda$. In position Fig. 34B it is a minimum. Apparently the permeance in b is larger than in a as the section of passage is larger. We must, however, bear in mind that the zigzag lines surround one-half of the secondary slots and induce in them an electromotive force which is opposite to the primary one. The permeance in Fig. 34B has the equivalent value $0\cdot062 \times t_1/\lambda$. As an average we find for a ratio of slots of 1 to 2, $T''_{\text{K}} = 0\cdot105 \times t_1/\lambda$.

If the ratio of primary to secondary slots is 1 to 3, the maximum is $0\cdot083 \times t_1/\lambda$ (Fig. 35A). The minimum of the permeance $0\cdot074 \times t_1/\lambda$ (Fig. 35B). The average is $T''_{\text{K}} = 0\cdot08 \times t_1/\lambda$.

The calculation is somewhat more complicated if primary and secondary slots have not an even ratio. If this ratio is 1 to 1·5, Figs. 36A and 36B give the distribution of lines in the minimum and maximum positions. The maximum of permeance is $0\cdot13 \times t_1/\lambda$, the minimum $0\cdot102 \times t_1/\lambda$, the average $0\cdot120 \times t_1/\lambda$.

Generally we find, for a ratio of slots 1 to y,

Maximum permeance $\frac{0\cdot25}{y} \times \frac{t_1}{\lambda}$.

Minimum permeance $\frac{0\cdot50}{y} \times \frac{(y-1)^2}{y} \times \frac{t_1}{\lambda}$.

Average value of the permeance $T''_{\text{K}} = \frac{1}{6} \times \left[\frac{1}{y} + \frac{(y-1)^2}{y^3}\right] \times \frac{t_1}{\lambda}$.

Instead of using this complicated expression, we will apply a simple formula which is sufficiently exact for practical purposes:

$$T''_{\text{K}} = \left(0\cdot038 + 0\cdot127 \times \frac{n_1}{n_2}\right) \times t_1/\lambda.$$

For further simplification, we put the permeance of zigzag leakage per centimetre of core length per slot

$$T''_K = C''_K \times t_1/\lambda. \quad \ldots \ldots \ldots \quad (8)$$

C''_K can be taken from the straight line in Fig. 37.

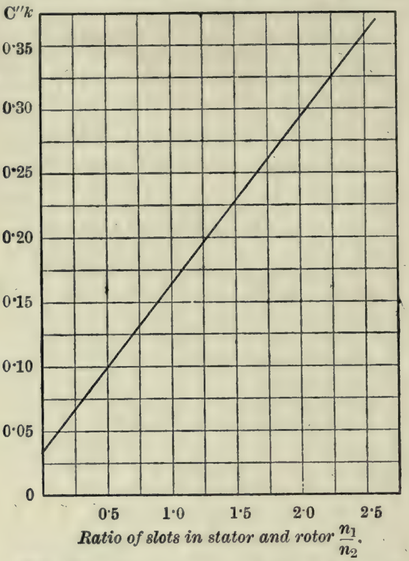

FIG. 37.—CONSTANT OF ZIGZAG LEAKAGE.

For the calculation of the zigzag leakage it is very important to take account of the opening of the slots, which we have neglected in our calculation hitherto. From Figs. 38A and 38B we deduce :—

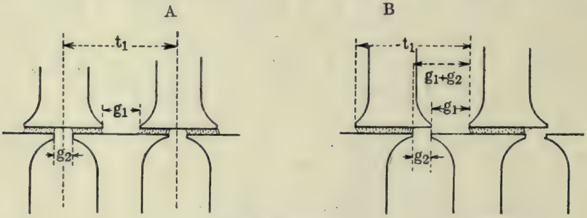

FIG. 38.

1. That the section for the zigzag lines has been reduced by an amount corresponding to the sum of primary and secondary slot openings.

2. That the time during which the zigzag leakage is zero has ceased to be a moment only (*see* Fig. 33A). It is an amoun equal to $0{\cdot}85 \times (g_1 + g_2)$ (Fig. 38B).

Representing graphically the permeance of the zigzag leakage path for $n_1/n_2 = 1$, we find the curve Fig. 39, curve I. referring to slots without openings, curve II. to slots with openings. The average value of the permeance is reduced by the openings in the ratio $\left[1 - \frac{0{\cdot}85 \times (g_1 + g_2)}{t_1}\right]^2$. Also, with other values of n_1/n_2 without slot opening, the permeance obtains only minimum values, but does not become zero. The latter can be caused by the slot openings only. With sufficient exactitude we can take account of the slot openings by multiplying the value of T_K'' calculated from formula (8) by

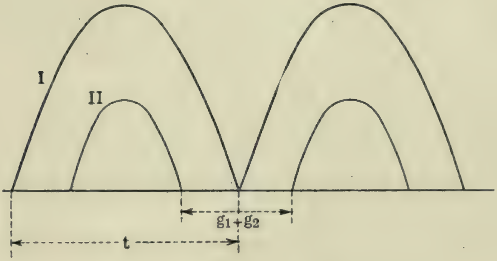

FIG. 39.

$$\left(1 - 0{\cdot}85 \times \frac{g_1 + g_2}{0{\cdot}5 \times (t_1 + t_2)}\right)^2. \quad . \quad . \quad . \quad . \quad (8A)$$

Continuing our example of p. 183, we put

Air-gap $\lambda = 1{\cdot}5$ mm.
Primary slots $n_1 = 72$.
Primary and secondary slot pitches $t_1 = 2{\cdot}5$ cm.
$t_2 = 3{\cdot}74$ cm.
Primary and secondary slot openings $g_1 = 0{\cdot}35$ cm., $g_2 = 0{\cdot}25$ cm.
Ratio of slots $n_1/n_2 = 1{\cdot}50$.

We find the permeance of zigzag leakage as follows :—

C_K'' (from Fig. 37) $= 0{\cdot}228$, and, therefore, from formula (8) $T_K'' = 0{\cdot}228 \times 2{\cdot}5/0{\cdot}15 = 3{\cdot}80$, uncorrected. Correction for the slot openings (8A),

$$\left(1 - 0{\cdot}85 \times \frac{0{\cdot}35 + 0{\cdot}25}{0{\cdot}5 \times (2{\cdot}5 + 3{\cdot}75)}\right)^2 = 0{\cdot}71.$$

Correct value of $T_K'' = 0{\cdot}71 \times 3{\cdot}8 = 2{\cdot}70$.

The permeance of the slot leakage was (p. 183) $T_K' = 3{\cdot}50$, so that slot leakage and zigzag leakage together give a total permeance per centimetre slot length of $T_{Kt} = T_K' - T_K'' = 3{\cdot}5 + 2{\cdot}7 = 6{\cdot}2$.

If we put the core length $l = 20$ cm., the number of wires per stator slot $= 9$, the frequency $= 50$, and assume that we have to deal with three phase current, we can calculate the E.M.F. of self-induction e_s' for 10 amperes short-circuit current due to the leakage in the iron only as follows :—

Slots per phase $z_1 = 72/3 = 24$.

Total permeance per phase $24 \times 20 \times 6{\cdot}25 = 3{,}000$.

Maximum number of lines of force of leakage with 10 amperes, 9 wires per slot : $1{\cdot}25 \times 9 \times 10 \times 1{\cdot}41 \times 3{,}000 = 0{\cdot}475 \times 10^6$, $e_s' = 4{\cdot}5 \times 9 \times 50 \times 0{\cdot}475 \times 10^6 \times 10^{-8} = 9{\cdot}61$ volts.

Naturally it was not necessary to go right back to the number of leakage lines. We might have calculated e_s' directly from a single formula

$$e'_s = 7{\cdot}7 \times i \times v \times \frac{S^2}{Z_1} \times T_{Kt} \times l \times 10^{-8} \text{ volts.} \quad . \quad . \quad (9)$$

With squirrel-cage rotors the zigzag leakage is somewhat smaller than formula (8) gives. Considering the case where $n_1/n_2 = 1/2$, and giving stator and rotor slots the relative position (Fig. 40), we see that the bars in the slots AA, which are opposite the face of a tooth, have a considerably larger self-induction than the bars BB.

In BB the mutual induction with the primary slots is prevalent. Therefore, we may assume that the bars AA carry almost no current at all, and that BB receive the full primary ampere-turns directly from the opposite stator slots. Therefore the zigzag leakage in this position of the rotor is practically zero.

If in Fig. 41 curve I. shows the permeance of the zigzag leakage dependent upon the position of the rotor for the wound rotor and for $n_1/n_2 = 1/2$, the curve for a squirrel-cage motor assumes approximately the form II. The ratio of the average values of permeance in both cases is 0·80.

We shall apply this factor generally, and shall calculate the leakage with squirrel-cage rotors by multiplying by 0·80 *the zigzag leakage found through the application of formula* (8).

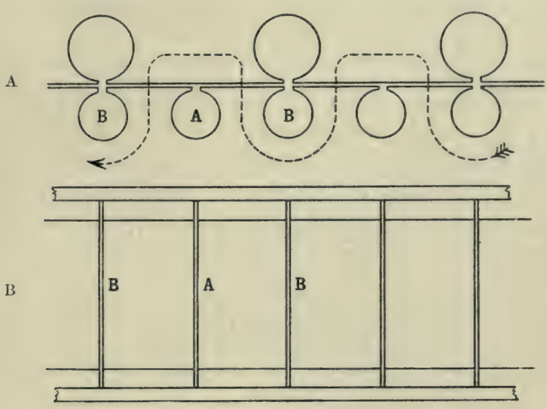

Fig. 40.

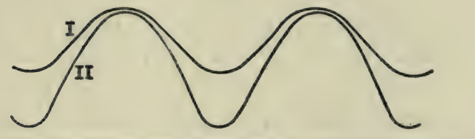

Fig. 41.

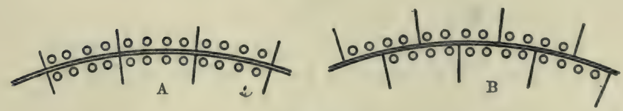

Fig. 42.

Also with wound rotors our formula only gives quite correct values for the relative position of rotor and stator in which the phases of both parts are exactly opposite one another (Fig. 42A). In other positions of the rotor the leakage of induction motors is somewhat larger, but only a very small percentage. With single-phase motors, where the alternations are larger than with any other class of induction motor, the permeance differs by 2 to 5 per cent. from the average value, according to the rotor position.

It remains to refer to the special conditions in the case that the span of the coils is shorter than the pole pitch, and the number of coils is equal to the number of slots. Fig. 43 shows the diagram of a winding for a three-phase machine with five slots per phase per pole, and a coil span which differs from the pole pitch by two slots.

The wires of one phase fill only $5-2=3$ slots per pole (Nos. 3, 4 and 5) completely, $2\times 2=4$ slots only half (Nos. 1, 2, 6 and 7); the latter contain also wires of the two other phases. At the moment when the current in the phase under consideration has attained its maximum, the current in the two others is just half as large. The leakage flux in the slots 3, 4 and 5 is exactly as large as it would be if coil span and pole-pitch were equal. In the slots 1, 2, 6 and 7 the leakage is less in the ratio $1{\cdot}5/2=0{\cdot}75$. The E.M.F. of self-induction for the three centre slots may be called e_s; then the corresponding value for the four side slots is $0{\cdot}75\times 0{\cdot}50=0{\cdot}375\times e_s$. The E.M.F. of self-induction per phase and pole becomes, therefore, $3\times e_s + 4\times 0{\cdot}375\times e_s=4{\cdot}5\times e_s$.

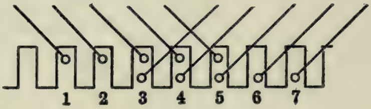

FIG. 43.

By the reduction of the coil span by two slots the leakage has been reduced in the ratio $4{\cdot}5/5=0{\cdot}90$. Generally speaking, the leakage of the stator of a three-phase motor with n' slots per phase and pole is reduced in the ratio $(1-0{\cdot}25\times a/n')$, if the coil span has been shortened by a slots. For two-phase and three-phase motors this relation is $(1-0{\cdot}50\times a/n')$. Naturally only one-half of the zigzag permeance is to be multiplied by these factors, and the formulæ are only to be used when

a is smaller than n'. The calculation of the leakage with any other conditions does not offer any difficulty and can be carried through in a quite similar way.

(C) *The Leakage of the Coil Ends.*

The leakage lines passing through the coil ends have their paths entirely or partly in air (Fig. 44); the larger portion issues from the stator and rotor teeth and closes through the end brackets or in a large circle through the air. A calculation of the coil end leakage is not impossible but very difficult, especially when there are several phases situated in different planes. The self-induction of the coil ends can be measured directly on test coils. Giving these coils different forms we may gain valuable figures for the leakage coefficients. On the other hand, we can also obtain good constants for our calculations by comparing test results on machines with the same winding but different core lengths or by calculating the leakage, in the iron body and comparing with the test.

I have carried out a simple test for *direct measurement* of the self-induction of coil ends which I will briefly describe :—

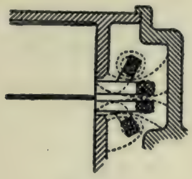

FIG. 44.

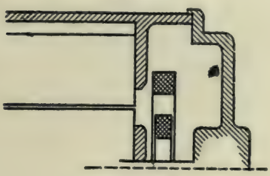

FIG. 45.

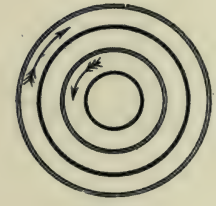

FIG. 46.

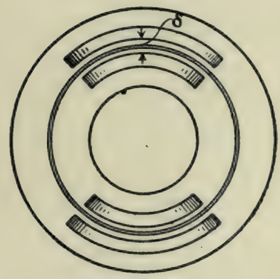

FIG. 47.

Two circular coils of the same section and equal number of turns but different diameter were introduced between the laminated iron and the end bracket of an induction motor (Figs. 45 and 46 show the arrangement). The two coils were connected one against the other. The permeance per centimetre average circumference was found to be 0·58 C.G.S. This value is of the same order as with the coil ends of completely wound motors.

The section which the leakage lines find for their passage is proportional to the mean lengths of the coil ends—*i.e.*, proportional to the rotor circumference; besides, this section is dependent on the average distance *d* between rotor and stator winding (Fig. 47). With the same class of winding there is very little difference in this amount from one machine to another, but the type of the winding itself has a considerable influence on the average distance of the coil ends of stator and rotor winding, and, therefore, on the permeance per centimetre circumference.

We will distinguish between the following classes of windings with special reference to their coil ends:—

1. Phase winding or chain winding.

Phases are wound separately; some groups of coils are bent up and are reaching over the others. The coils of the different groups are wound concentric—*i.e.*, the coils are lying in slots 5 and 10, 4 and 9, &c. (Fig. 48).

(*a*) The number of coil groups per phase is equal to half the number of poles (Fig. 48).

(*b*) The number of groups per phase is equal to the number of poles (Fig. 49).

2. The single-phase winding with concentric flat coils. Number of groups equal to number of poles (Fig. 50).

3. All coils have the same form, and are wound in such a way that the winding is perfectly symmetrical. The number of coils is either equal to the number of slots or equal to one-half or a multiple of it.

(*a*) Disc winding or involute winding. The coil ends are bent up so that the chief dimension is vertical to the axis of the machine (Fig. 51).

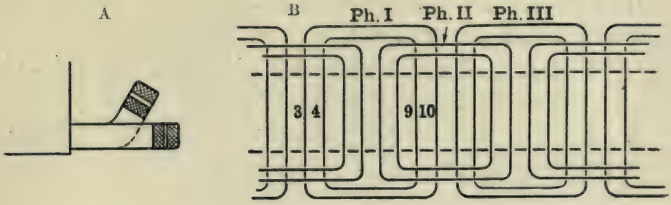

FIG. 48.

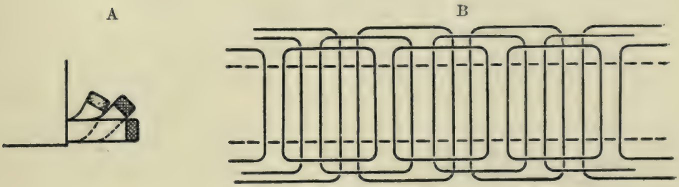

FIG. 49.

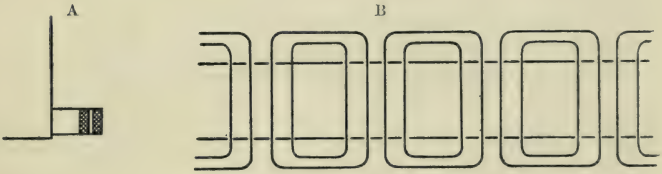

FIG. 50.

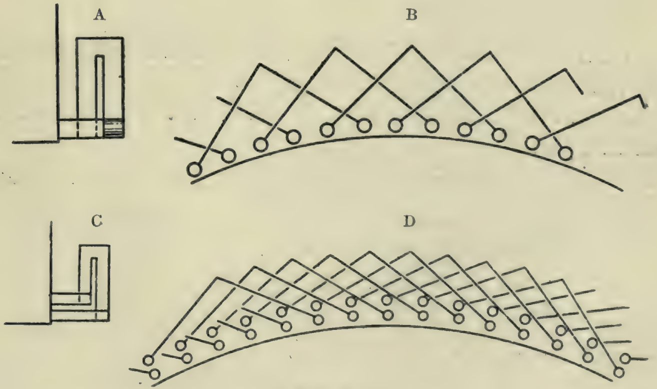

FIG. 51.

(*b*) Cylindrical winding, diamond coils. The coils are flat and form a cylinder (Fig. 52).

(*c*) Short type of winding or mush winding. The coil ends occupy the smallest possible space (Fig. 53).

4. For the rotor we have to add: The short-circuit rotor with its different forms (Figs. 54A to 54E).

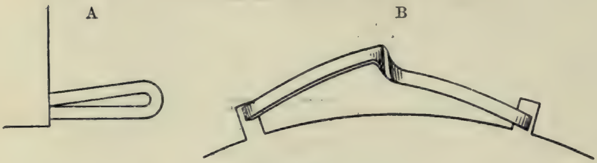

FIG. 52.

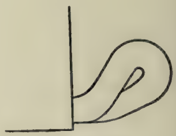

FIG. 53.

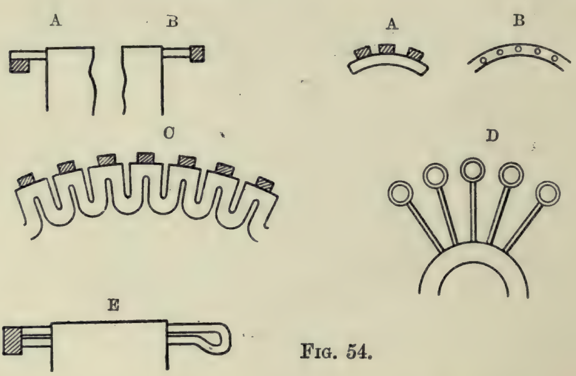

FIG. 54.

With the same class of winding the ampere-turns embodied in the coil ends are proportionate to the ampere-turns per phase and pole. We, therefore, put the E.M.F. of self-induction for this part

$$e_s'' = C_K'' \times d \times \pi \times i \times \nu \times (n_1' \times s)^2 \times 10^{-8} \text{ volts.} \quad . \quad (10\text{A})$$

n_1'=Number of slots per phase and pole primary.
d=Rotor diameter in centimetres.
s=Number of wires per slot.
i=Current per phase.

Putting $S = z_1 \times s$ and $z_1 = p \times n_1'$,

$$e_s'' = C_K''' \times d \times \pi \times i \times \nu \times \frac{S^2}{p^2} \times 10^{-8} \text{ volts.} \quad . \quad (10\text{B})$$

We shall use this formula independent of the number of wires contained in the coil ends—*i.e.*, regardless of its being $n_1' \times s$, or a fraction or a multiple of it, and regardless of our dealing with a three, two or one-phase winding. The constant C_K''' which I have developed for the different types of winding is not only to take account of the permeance, but also of the ratio between the number of wires in the coil ends to the number of wires per phase and pole.

The leakage constant for the coil ends is to be taken from the following Table VII.

Table VII.—*Leakage Constant of the Coil Ends,* C_K'''.

Winding of the rotor.	Winding of the Stator.											
	No. 1, chain winding.			No. 3a, disc winding.			No. 3b, cylindrical winding,			No. 3c, short type winding.		
	1a & 1b.	1b.	1b (2).									
	3 ph.	2 ph.	1 ph.	3 ph.	2 ph.	1 ph.	3 ph.	2 ph.	1 ph.	3 ph.	2 ph.	1 ph.
No. 1a & 1b, chain	8·7	6·0	3·1	...	...	...	...	...	...	...	...	...
No. 3a, disc.	8·7	6·0	3·1	9·0	6·1	4·5	5·0	3·5	2·5	...	...	...
No. 3c, short type	...	...	...	...	...	...	...	...	...	5·8	4·0	2·9
No. 4a & 4b, short-circuit rotors.	6·1	4·2	2·5	4·5	3·2	2·5	2·5	2·1	1·5	4·15	2·9	2·3

The values for the leakage constant of the coil ends C_K''', as given by Table VII., constitute, perhaps, the most important part of this study. These values have been obtained from several hundreds of picked and reliable test results with the various classes of machines and types of windings. It would be of little use to the reader if I gave a table of all these test results; a general survey of the results will suffice.

We notice first of all how small the coil-end leakage is *with short-circuit rotors.* In some cases it is only half as much as with wound rotors. This clearly is due to the reduced distance between rotor and stator windings at the coil ends (*d* in Fig. 47). Those types of short-circuit rotors where the bars are not short-circuited by simple rings, and where the distance between the two windings is much larger, show considerably worse results. The type Fig. 54C, for instance, the rings of which have zigzag form for the purpose of increasing their resistance, show, with three-phase cylindrical winding in the stator, a coil-end leakage constant $C_K''' = 5$, compared with 1·7 with the types Fig. 50A and 50B. Type 54D has with three-phase chain winding in the stator a $C_K''' = 9{\cdot}4$ instead of 6·1. A special class of rotor windings is formed by type Fig. 54E having a proper winding on one end, all bars being short-circuited by a ring on the other end. With three-phase chain winding in the stator we obtain a $C_K''' = 7{\cdot}8$.

The *two classes of three-phase chain winding, types* 1A *and* 1B, have been treated as perfectly equal as regards leakage, though the number of wires in the coil ends in case 1A is double that in case 1B (Figs. 55A and 55B), provided that we consider one phase only. If, however, we calculate the mean number of ampere-turns which produce lines round the coil-ends of one phase at the moment when the current of this phase is at its maximum, the two others being half as large, we find the same average ampere-turns in both cases.

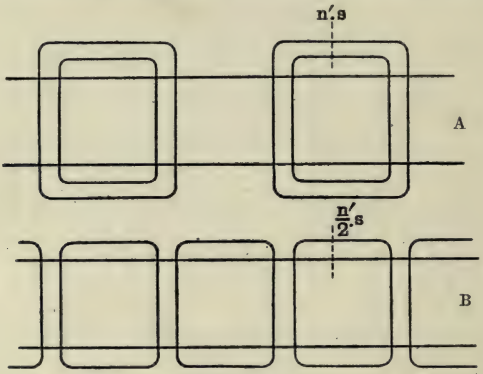

FIG. 55.

The constants for *three-phase current* are about 40 to 50 per cent. higher than those for *two-phase current.* This is due to the mutual induction between the three phases in the case of a three-phase winding.

As the *most favourable class of winding for the stator the cylindrical winding is prominent.* The leakage of the coil-ends is almost only one-half of that with chain winding. We can, however, improve the values for the cylindrical winding still more if we reduce the span of the coils. If β denotes the angle of reduction of coil span (*see* p. 168), the correction in C_K''' is approximately cos $\beta/2$. For instance, with three-phase cylindrical winding and $\beta = 60$ deg. and short-circuit rotor we obtain $C_K''' = 2{\cdot}5 \times \cos(30 \text{ deg.}) = 2{\cdot}5 \times 0{\cdot}86 = 2{\cdot}15$.

The constants given in Table VII. naturally represent only average values. With machines with the same class of windings which are also otherwise identical, C_K''' varies from 10 to 15 per cent., due to differences in workmanship. With the same type of winding but with different numbers of poles and different arrangements C_K''' varies about 25 per cent. If there is no *exceptional arrangement* of the coil ends the mistake we can make in the calculation of the total short-circuit current when applying our constants is not greater than about 5 or 6 per cent. As an instance of an "exceptional arrangement," we may mention the very tight enclosure of the coil-end by the end brackets.

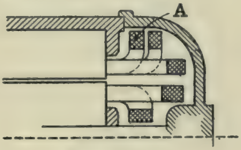

FIG. 56.

With a completely enclosed coal-cutter motor with chain winding and wound rotor, where the unusually thick end brackets were practically touching the winding all round (Fig. 56), $C_K''' = 13$ instead of $8{\cdot}7$ has been observed. The large diameter of the bearing shells almost touching the rotor winding also contributed to increase C_K'''.

By going a little more into details in this case I meant to show what factors may increase the coil-end leakage, so that one may take account of them approximately. Of far worse influence than placing iron close to the winding is the introduction of magnetic bodies between stator and rotor winding. A three-phase machine with disc winding and short-circuit rotor showed $C_K''' = 4\cdot5$, in agreement with our Table. To protect the rings of the rotor short-circuiting the bars against centrifugal force an iron ring, F, as shown by Fig. 57, was fitted. Through its influence C_K''' became 7—*i.e.*, the coil-end leakage had been increased 55 per cent. This meant actually about 25 per cent. reduction of the starting torque of the machine, together with 2 or 3 per cent. reduction in the power factor.

On the other hand, a favourable influence is experienced if the magnetic path for the coil-end leakage is given a high resistance. For instance, with machines having no end brackets, but pedestal bearings instead, comparatively low leakage coefficients are observed. The $C_K''' = 8\cdot5$ of a three-phase machine with phase winding (chain winding) in stator and rotor has been reduced to 6·8 by removing the end brackets.

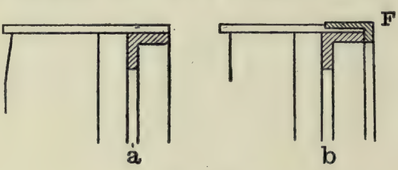

FIG. 57.

IV.—The Leakage Factor.

We have put the leakage factor $\tau = i_0 / i_K'$, and will now introduce the values for i_0 and i_K', which have been developed in the previous chapters. According to formula (6) *the no-load current* to cover the air magnetisation is

$$i_{0l} = C_0' \times \frac{e}{\nu} \times \frac{p^2}{S^2} \times \frac{\lambda}{Q_l} \times 10^8.$$

With machines with partly-closed slots, the no-load current i_{0f} for the iron magnetisation is about 20 per cent., with open slots about 10 per cent. of the magnetising current for air only. Naturally, this value alters from case to case. We will introduce an i_{0f} equal to $0\cdot10 \times i_{0l}$, and will make a corresponding correction if necessary. We then find

$$i_0 = 1\cdot1 \times C_0' \times \frac{e}{\nu} \times \frac{p^2}{S^2} \times \frac{\lambda}{Q_l} \times 10^8.$$

Introducing the value for Q_l and putting in round figures $Q_l = d \times \pi \times l \times 0{\cdot}85 = 2{\cdot}67 \times d \times l$ (the coefficient 0·85 has been introduced with regard to the slot openings, and may be corrected if necessary), we find

$$i_0 = 0{\cdot}40 \times C_0' \times \frac{e}{\nu} \times \frac{p^2}{S^2} \times \frac{\lambda}{d \times l} \times 10^8.$$

Adding *slot leakage* and *zigzag leakage*, we can put, according to formula (9),

$$e_s' = 7{\cdot}7 \times i_K' \times \nu \times \frac{S^2}{z_1} \times T_{Kt} \times l \times 10^{-8}.$$

According to formula (10B), *the coil-end leakage* E.M.F. is

$$e_s'' = C_K''' \times d \times \pi \times i_K' \times \nu \times \frac{S^2}{p^2} \times 10^{-8}.$$

We add both these E.M.F.s, so that $e = e_s' + e_s''$; thus

$$e = \nu \times i_K' \times S^2 \times 10^{-8} \times \left(\frac{7{\cdot}7 \times l + T_{Kt}}{z_1} + C_K''' \times \frac{d \times \pi}{p^2}\right),$$

or

$$i_K' = \frac{e \times 10^8}{\nu \times S^2 \times \left(\frac{7{\cdot}7 \times l \times T_{Kt}}{z_1} + C_K''' \times \frac{d \times \pi}{p^2}\right)}.$$

Forming now $\tau = i_0/i_K'$, we obtain

$$\tau = (3{\cdot}05 \times C_K''' \times T_{Kt}) \times \frac{p^2}{z_1} \times \frac{\lambda}{d} + (0{\cdot}40 \times \pi \times C_0' \times C_K''') \times \frac{\lambda}{l}, \quad (11\text{A})$$

$$\tau = K_1 \times \frac{p^2}{z_1} \times \frac{\lambda}{d} + K_2 \times \frac{\lambda}{l}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11\text{B})$$

With this formula for τ we have to bear in mind that the iron resistance has been introduced with 10 per cent. of the air resistance, and that the ratio between the true air section and the apparent air section has been put equal to 0·85. If these figures are different, τ changes proportionately. From the two parts of formula (11B) the first only is dependent on the leakage in the slots, the second only on the leakage through the coil ends.

To clearly understand the formula we will introduce practical average values for the constants. An average figure for T_{Kt} *with partly closed slots* of 25 mm. to 35 mm. height and 9 mm. to 12 mm. width is about 5·5 (*see* p. 192 and Fig. 25). *If the rotor has many slots, and if the stator slots are broad and* *shallow*, T_{Kt} *goes down to about* 4·5. With *few rotor slots and narrow and deep stator slots and with very small air-gap*, T_{Kt} *can go up to* 6·5, in very favourable cases even to 8. When first laying out a machine, the size of slots, &c., not being yet known, *we may start with* $T_{Kt}=5·5$ and correct it afterwards. *With open slots* T_{Kt} varies between 3·5 and 4·5. $T_{Kt}=4$ *is a good average* for a preliminary calculation.

With three-phase motors and chain winding in stator and rotor we can, according to Table VII., put $C_K'''=8·7$. C_0' for three-phase currents will be equal to 0·43 (p. 174).

Introducing these values we find (formula 11) that $K_1=3·05\times0·43\times5·5=7·2$. K_1 *varies with partly closed slots between* 6 *and* 8·5; *with entirely open slots between* 4·5 *and* 6.

We find further that $K_2=0·40\times\pi\times0·43\times8·7=5·1$, say 5·0. Therefore, as an average,

$$\tau=7·5\times\frac{p^2}{z_1}\times\frac{\lambda}{d}+5·0\times\frac{\lambda}{l}. \quad . \quad . \quad . \qquad (11\text{C})$$

Taking into account the fact that with

Three-phase current $K_2=0·57\times C_K'''$,
Two-phase current $K_2=1·2\times C_K'''$,
Single-phase current $K_2=2·4\times C_K'''$,

the coil-end leakage constant K_2 can be very easily calculated by means of Table VII.

As an example, we will calculate the leakage coefficient for a three-phase motor with chain winding in the primary and secondary having a rotor diameter $d=50$ cm., a core length $l=20$ cm., an air-gap $\lambda=1$ mm. We find for the preliminary calculation (formula 11C)

$$\tau=7·5\times\frac{p^2}{z_1}\times\frac{0·10}{50}+5·0\times\frac{0·10}{20},$$

$$\tau=0·015\times\frac{p^2}{z_1}+0·025.$$

Introducing for the number of slots per phase the figure $z_1=48$, we find

$$\tau=\frac{p^2}{3,200}+0·025.$$

For different numbers of poles with the same number of slots we may calculate the following table :—

$p = 2 \qquad \tau = 0{\cdot}0013 + 0{\cdot}025 = 0{\cdot}026 \qquad \tau/p = 0{\cdot}0130$

$p = 4 \qquad \tau = 0{\cdot}005 + 0{\cdot}025 = 0{\cdot}030 \qquad \tau/p = 0{\cdot}0075$

$p = 6 \qquad \tau = 0{\cdot}011 + 0{\cdot}025 = 0{\cdot}036 \qquad \tau/p = 0{\cdot}0060$

$p = 8 \qquad \tau = 0{\cdot}020 + 0{\cdot}025 = 0{\cdot}045 \qquad \tau/p = 0{\cdot}0056$

$p = 12 \qquad \tau = 0{\cdot}045 + 0{\cdot}025 = 0{\cdot}070 \qquad \tau/p = 0{\cdot}0058$

This table will serve to check Behrendt's formula. We, find that with an exactness of ± 14 per cent. we can put for 4, 6, 8 and 12 poles $\tau = 0{\cdot}0066 \times p$, or, as average for 6, 8 and 12 poles, $\tau = 0{\cdot}0058 \times p$, in accordance with Behrendt's formula, which claims that τ reverses proportional to the pole-pitch—*i.e.*, with the same motor proportional to the number of poles.

The formula (11) which we developed shows that Behrendt's formula is not correct, but the fact that the formula for a given carcase consists actually of one part proportional to p and a constant, makes τ appear directly proportional to the number of poles p.

Taking the formula of Behrendt in this fashion, it takes account of all classes of leakage, including the coil-end leakage. Behrendt's constant of proportionality is dependent on the core width l only through the coil-end portion of equation (11B), so that one may apply the formula

$$\tau = \text{constant} \times \text{air-gap/pole-pitch},$$

if one is prepared to allow ± 40 per cent. variation in the constant. Applying this formula on our example,

$$\tau = 0{\cdot}0066 \times p = \text{constant} \times \frac{\lambda \times p}{d \times \pi}.$$

Therefore, the constant is 10·5, a value lying within the limits given by Behrendt.

To study the influence of an *alteration of the number of slots* z_1 we have to look at the complete formula (11A). If with one and the same motor we increase z_1 we have to make the slots narrower. Consequently, the permeance for the slot leakage will be increased, but the permeance of the zigzag leakage will be reduced. The ratio T_{Kt}/z_1 which occurs in the first part of formula (11A) decreases with increasing z_1. Naturally the number of slots is very rarely determined by the leakage only. Heating, arrangement of wires and insulation, as well as existing slot dies, are of the greatest importance, and it would lead us much too far away from our object if we were to introduce these factors into this study of the subject.

We will now *compare three-phase, two-phase and single-phase motors.*

If the three-phase motor in the example we have just calculated is changed into a two-phase motor by altering the winding, the number of slots being the same, the number of slots per phase and pole is increased in the ratio $3/2 = 1{\cdot}5$. C_0' becomes 0·88 instead of 0·43 (*see* p. 174)—*i.e.*, the first part of formula (11B), which depends on slot and zigzag leakage only, becomes $\frac{1}{1{\cdot}5} \times \frac{0{\cdot}88}{0{\cdot}43} = 1{\cdot}4$ times as large as with the three-phase motor.

K_2 increases in the ratio of

$$\frac{0{\cdot}88}{0{\cdot}43} \times \frac{C_K''' \text{ for two-phase}}{C_K''' \text{ for three-phase}} = \frac{0{\cdot}88}{0{\cdot}43} \times 0{\cdot}68 = 1{\cdot}4,$$

i.e., with the same iron body a two-phase motor has a leakage factor 40 per cent. larger than with the three-phase motor. Utilising Tables I. and II. we find the following relations between the maximum power factor of three-phase and two-phase motors :—

$\cos \phi_{\max}$ with three-phase winding.	Corresponding $\cos \phi_{\max}$ with two-phase winding.
0·90	0·87
0·85	0·805
0·80	0·74

i.e., the power factor of a two-phase motor is 0·03 to 0·06 lower than that of the corresponding three-phase motor.

With single-phase motors K_1 becomes twice as large as with three-phase motors. K_2, too, alters in the same ratio, except with chain winding in the stator and with machines with any other winding in the stator but with a rotor of the squirrel-cage type. With single-phase winding in the stator the bent coils do not come in, so that here $\frac{C_K''' \text{ for three-phase}}{C_K''' \text{ for two-phase}}$ becomes larger than 2—viz., $8{\cdot}7/3{\cdot}1 = 2{\cdot}8$.

With short-circuit rotors (except for chain winding in the stator) this relation is 1·75, therefore *no definite relation between the leakage factor of a three-phase motor and a single-phase motor exists. The class of winding and the ratio of core lengths to rotor diameter comes into consideration.*

If we interrupt one phase of a three-phase motor running at no load without changing the voltage and frequency the no-load current increases in the ratio $\frac{i_0 \text{ one-phase}}{i_0 \text{ three-phase}} = 1{\cdot}70$. For the short-circuit current we find as an average about $\frac{i_K \text{ one phase}}{i_K \text{ three phase}} = 0{\cdot}81$. These figures have been gained from a considerable number of practical observations. They are very convenient if one wishes to design a single-phase machine, a three-phase machine being given.

As an average value we put $\frac{\tau \text{ for one-phase}}{\tau \text{ for three-phase}} = \frac{1{\cdot}70}{0{\cdot}81} = 2{\cdot}1$. Then we may give the approximate relations between the maximum power factor of a single-phase motor and a three-phase motor in form of a table :—

Cos $\phi_{\max}$. with three-phase winding.		Corresponding cos $\phi_{\max}$. with one-phase winding.
0·90	....	0·82
0·85	....	0·74
0·80	....	0·67

Generally the power factor of a single-phase motor with normal load is smaller than the maximum power factor, and, therefore, the conditions with a single-phase motor are even worse than our table shows.

V.—General Example.

Three-phase Motor.—50 periods, 6 poles, 1,000 volts. Phases star connected.

$1{,}000/1{\cdot}73 = 578$ volts per phase.

Slip-ring rotor. Chain winding in stator, disc winding in rotor.

Dimensions: Rotor diameter, $d = 40$ cm.; circumference $= 40 \times \pi = 126$ cm.

Core length, $l = 16$ cm.

Air-gap $\lambda = 0{\cdot}10$ cm.

Number of slots, $n_1 = 72$; $z_1 = 24$ slots per phase
$n_2 = 82$.

Slot heights, $h_1 = 3{\cdot}25$ cm.; $h_2 = 2{\cdot}5$ cm.

Slot widths, $b_1 = 1{\cdot}05$ cm.; $b_2 = 0{\cdot}8$ cm.

Slot openings, $g_1 = 0{\cdot}2$ cm.; $g_2 = 0{\cdot}2$ cm.

Slot ratio, $n_1/n_2 = 72/82 = 0{\cdot}88$.

Sum of all slot openings—fringe, stator $0{\cdot}85 \times n_1 \times g_1 = 12$ cm.
rotor $0{\cdot}85 \times n_2 \times g_2 = 16$ cm.

Rotor circumference—slot openings $= (126 - 12) \times 110/126 = 100$ cm. (*see* p. 191).

Air section $100 \times 16 = 1{,}600$ cm.^{2}
Tooth section: Stator 730 cm.^{2}; rotor 680 cm.^{2}
Wires per stator slot $s = 9$.

1 *Preliminary Figure for Leakage Factor* τ. From formula (11c)

$$\tau = 7{\cdot}5 \times \frac{6^2}{24} \times \frac{0{\cdot}10}{40} + 5 \times \frac{0{\cdot}10}{16} = 0{\cdot}028 + 0{\cdot}031 = 0{\cdot}059,$$

say 0·065, because we require more tooth ampere-turns than allowed for in formula 11.

2. *No-load Current.*

Formula (3B):

$$\text{Air density } B_l = \frac{0{\cdot}74 \times 578 \times 6 \times 10^8}{24 \times 16 \times 50 \times 1{,}600} = 8{,}300 \text{ C.G.S.}$$

Stator tooth density $B_t = 8{,}300 \times 1{,}600/730 = 18{,}200$ C.G.S.

Ampere-turns for air $8{,}300 \times 0{\cdot}80 \times 0{\cdot}10 \times 2 = 1{,}320$
„ „ teeth (Table IV.) ....= 200
„ „ yokes ....= 80—1,600

No-load Current (formula 5A) $i_0 = \dfrac{1{,}600 \times 0{\cdot}38}{24 \times 16} = 9{\cdot}5$ amperes.

3. *Leakage Permeances.*

(*a*) Slot leakage.

Primary slots (from Fig. 25) $= 1{\cdot}95$
Secondary slots (from Fig. 25) $1{\cdot}85 \times \dfrac{n_1}{n_2} = 1{\cdot}70$
Permeance of slot leakage: $T_K' = 3{\cdot}65$

(*b*) Zigzag leakage. C_K'' (from Fig. 37)......0·15

From formulæ (8) and (8A): $\dfrac{100^2 \times 0{\cdot}15}{126 \times 72 \times 1{\cdot}0} = 1{\cdot}65$

Total leakage permeance per cm. slot $T_{Kt} = 5{\cdot}30$ cm.

Ditto for 24 slots: $5{\cdot}30 \times 24 \times 16 = 2{,}050$ cm.

Slot Leakage + Zigzag Leakage.—Voltage per phase per 1 ampere short-circuit current (formula 9): $2{,}050 \times 8 \times 50 \times 16^2$ ………………………………$= 2{\cdot}10$ volts

Coil End Leakage.—Voltage per phase per 1 ampere short-circuit current ($C_K''' = 8{\cdot}7$ from Table VII.) (formula 10) $8{\cdot}7 \times 126 \times 16^2 \times 50 \times 10^{-8}$ ………$= 2{\cdot}25$ volts

Total Leakage.—Voltage per phase per 1 ampere short-circuit current ………………………………$= 4{\cdot}35$ volts

Table VIII.—*Showing the Effect of Altering the Slot Openings and the Class of Winding.*

Slot opening g.		E.M.F. of leakage per phase for 1 amp. short-circuit current.				Short-circuit current. Amps.	No-load current. Amps.	Leakage factor. τ'.	Cos ϕ. max.
Stator g_1.	Rotor g_2.	Slot leakge Volts.	Zigzag leakge Volts	Coil end leakge Volts.	Total leakge Volts.				
0·1cm.	0·1cm.	1·85	0·83	2·25	4·93	117 0	8·55	0·073	0·872
0·2 ,,	0·2 ,,	1·45	0·65	2·25	4·35	133·0	9·5	0·071	0·875
0·3 ,,	0·3 ,,	1·36	0·49	2·25	4·10	142·0	10·65	0·075	0·870
Practically open slots	0·25 ,,	1·36	0·16	2·25	3·77	153·0	12·7	0·083	0·858
Do.	Do.*	1·36	0·16	1·16	2·68	215·0	12·7	0·059	0·90

* Chain winding replaced by cylinder winding $C_K''' = 4{\cdot}2$ (Table VII.)

4. *Short-circuit current* for $\cos\phi_K = 0{\cdot}2$

$$i_K' = 578/4{\cdot}35 \text{ volts} \times 1 = 133 \text{ amperes.}$$

5. *Leakage factor* $\tau = i_0/i_K' = 9{\cdot}5/133 = 0{\cdot}071$.

Maximum power factor, from Table I.: 0·875.

If the resistances are known, we calculate watts lost on short-circuit. In our case, $\cos\phi_K = 0{\cdot}35$.

Short-circuit current $i_K = i_K' \times \sin\phi_K = 133 \times 0{\cdot}94 = 125$ amps.

$$\tau' = 9{\cdot}5/125 = 0{\cdot}076.$$

$\cos\phi$ normal, &c., from Table I.

Maximum output of the motor (formula 1).

$$\frac{1{,}000 \times 1{\cdot}73 \times 125 \times 0{\cdot}37}{746} = 108 \text{ B.H.P.}$$

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